Predation and Sectorial Composition throughout the Transition*

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Abstract

Successful development processes involve "sectorial structural change", shifts of factors between different productive sectors, but also, "institutional structural change", shifts of factors from unproductive activities (predation) to productive ones. This paper analyzes the feedback process between the sectorial and the institutional structural changes in a model in which the labor share in agriculture is lower than in other sectors. Along the transition sectorial structural change emerges: employment in agriculture declines. Consequently, total labor share increases, raising the reward for working (while discouraging predation) and so, fostering institutional structural change. This, in turn, encourages capital accumulation, promoting sectorial structural change. This feedback mechanism widens differences in productivity and institutions among countries. Whereas zero-cost policies aimed to build institutions have positive effects, costly policies have uncertain effects due to complex feedback mechanisms.

Keywords: Growth Theory, Development, Sectorial structural change, Institutional structural change

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1 Introduction

Successful development processes are characterized by a structural change, commonly understood as a reallocation of factors from the primary sector in favor of other productive sectors, like manufacturing and services. We claim in this paper that a successful development process does not only involve a reallocation of factors across different productive sectors, but also and more importantly, a reallocation of factors from wasteful and unproductive activities to productive ones. Furthermore, we push up the idea that the "sectorial structural change", defined as a reallocation of factors across different productive sectors, affects incentives that agents have to devote time to unproductive activities. This implies the emergence of a feedback process between sectorial structural change and "intuitional structural change", defined as a reallocation of factors from unproductive to productive activities. This feedback process may shed some light on the difficulties of many developing countries to achieve a successful development process (see Quah, 1996, 1997 and Parente and Prescott, 1993).

Sectorial structural change has long been documented after the pioneering works by Kuznets (1966) and Chenery et al. (1986). A recent report of the World Bank (2014) documents that total employment in agriculture is about 48.5% in low and middle-income countries and only 5.4% in high-income ones. These numbers imply that the percentage of workers in agriculture in developing countries is almost 10 times the percentage in developed ones. Institutional structural change has recently received also a lot of attention, since it has been observed that the weight of wasteful and unproductive activities is larger in low income countries than in developed ones. This is considered by many economists the key factor to understand the failure of many developing countries to achieve a successful development process (see Rodrik et al., 2004, Besley and Persson, 2011, Acemoglu and Robinson, 2012).

We observe that resources in economies are devoted to both productive activities (resources are used in the production of goods and services) and unproductive activities (resources are used to generate returns but not goods). Since unproductive activities are characterized for being profitable, but wasteful, we will call them predation from now on. More precisely, we consider that predation is any activity in which an agent, acting as a predator, uses factors to capture the production generated from others, the preys. Examples of predation are: property crime, fraud, corruption, begging, lobbying, rent-seeking, etc. However, note that predation does not necessarily have to be illegal[1]. Predation can be derived from illegal activities (like property crime, burglary, corruption, etc.) but also from legal activities. For instance, lobbying, which is legal in many countries, implies the use of resources to press the government in order to get rewards, without producing any good or service. Rent-seeking is also another example of legal predation. Rent-seeking activities generate rents to the predators but not to the society (Acemoglu, 1995). In this group of rents, for instance, are included monopolies rents, since many economists identify monopolies as rent-seekers (Acemoglu and Robinson, 2019). Other authors even suggest that part of lawyers' activities might be considered predation (Murphy et al., 1991).

In this paper we push forward the idea that sectorial structural change and institutional structural change are deeply interconnected. The link between these two phenomena is related with the factorial income distribution: sectorial composition affects the labor share of the economy which is a key factor to determine the rewards for devoting time to predation or to production. Thus, sectorial composition drives the incentives scheme in the decision of time allocation. On the other hand, institutional structural change towards productive activities implies a reallocation of labor to these activities and promotes capital accumulation, generating sectorial structural change. Consequently, sectorial structural change greatly determines institutional structural change and vice versa.

A key feature of the model affecting the link between sectorial and structural changes is the fact that agriculture has a lower labor share (on income) than other sectors in the economy. At this respect, many recent studies have found agriculture to have lower labor share than both industry and services, while the capital share is similar for all sectors. Echevarria (1998) finds that in Canada the labor share represents 41% of value added in agriculture, 59% of value added in industry and 51% of value added in services. More recently, Valentinyi and Herrendorf (2008) find that while the agriculture shows the highest land share in US, around 11% of the value added in agriculture and less than 0.5% in the remaining sectors; the capital share is similar among sectors, it is 31% of value added in agriculture, 33% in manufactured consumption and 35% in services. Thus, the figures of land share and capital share together imply that the labor share in agriculture is lower than in

[1]Similarly, note that not all illegal activities are predation. Many illegal activities, such as homicides, prostitution or drug dealing cannot be defined as predation.

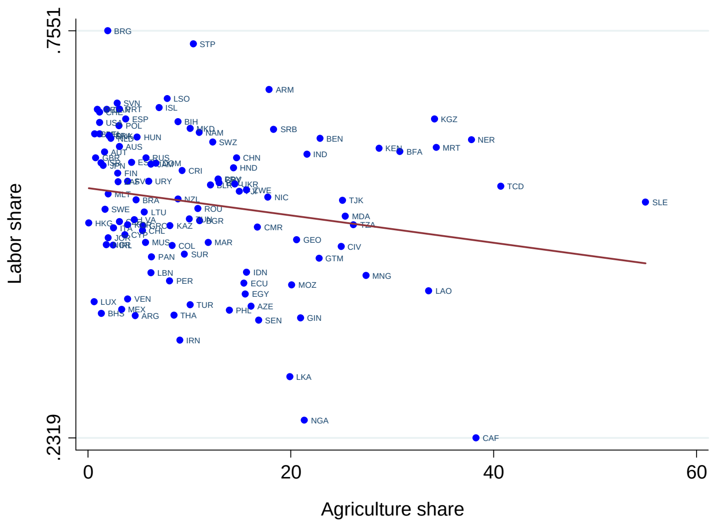


Figure 1: Source: Labor share is obtained from PWT 9.1 and the agriculture share over GDP is obtained from the World Bank Metadata Indicators.

other sectors due to the land intensity of agriculture compared with other sectors. This implies that developing countries having larger agriculture sectors should also have lower aggregate labor shares. In this respect, Figure 1 shows the relationship between labor share and agriculture share over GDP for a wide sample of developing and developed countries for the 2000s. We observe a clear negative relationship, with developed countries showing large labor shares and small agriculture sectors.

This paper proposes a mechanism that connects the three empirical facts mentioned above: *(i)* the higher amount of resources devoted to predation in developing countries, *(ii)* the higher portion of labor devoted to agriculture in developing countries and, *(iii)* the lower labor share in agriculture when compared to other sectors. We present a three-sector neoclassical growth model, namely agriculture, manufacturing and predation. The model has some key features related to the above empirical facts. As agricultural goods satisfy primary necessities, a "food problem" arises: a subsistence level of agricultural goods is required before the consumption of manufactured goods takes place. Thus, low income countries devote a higher portion of their resources to agriculture than high income countries (empirical fact *ii*). To be consistent with the empirical evidence provided by Valentinyi and Herrendorf (2008), it is assumed that the capital share is similar in agriculture and manufacturing, but agriculture is more land intensive than manufacturing, which implies that the agricultural sector shows a lower labor share than the manufacturing one (empirical fact *iii*). These two features together (the higher weight of agriculture in the production of low income countries and the lower labor share in agriculture), imply that the aggregate labor share is lower in poor countries than in rich countries (see Figure 1). Low labor shares in poor countries imply low reward for working relative to predation, discouraging work in productive activities and encouraging predation (empirical fact *i*). Thus, our model is consistent with the empirical evidence: low income countries are characterized by high weights of agriculture in production, low labor shares and high levels of predation. Insofar a country accumulates capital and subsistence needs lose weight in households' budgets, a sectorial structural change occurs in which more resources are devoted to producing manufactured goods. This sectorial structural change implies an increase in the weight of the manufacturing sector, which gives rise to an increasing aggregate labor share during the transition to the steady state. This generates an institutional structural change: the increase in labor share raises the relative reward for working in productive activities, which encourages the reallocation of labor from predation to production. Summarizing, when per capita capital is lower than the steady state level a sectorial and an institutional structural changes occur throughout the transition, in which predation falls and the weight of agriculture declines in favor of manufacturing.

This paper also contributes to understanding the differences in total factor productivity (TFP from now on) among countries and to explain why the differences in per capita income among countries have remained stable. It is widely accepted that differences in TFP are one of the main sources of differences in per capita income[2]. This paper proposes a mechanism that involves the reallocation of resources from predation to productive activities that amplifies differences in TFP: when productivity (in manufacturing or agriculture) rises, sectorial structural change arises: there is a shift in labor from agriculture to manufacturing, increasing the labor share and, the reward for working. This reduces predation, generating institutional structural

[2]See Easterly and Levine (2001), Hall and Jones (1999), and Parente and Prescott (2000).

change. Furthermore, institutional structural change increases the return to savings and encourages capital accumulation, which generates further sectorial structural change, fostering further capital accumulation. Thus, the feedback mechanism between sectorial and institutional structural changes and capital accumulation amplifies the effect of the improvement in technology. Thus, the results of our model are in line with the empirical research that emphasizes the differences in "social infrastructure", using the terminology by Hall and Jones (1999), in explaining differences in per capita income.

Thus, what Hall and Jones (1999) called "social infrastructure" is endogenous in our model and evolves along transition. In this respect, recent empirical studies such as Glaeser et al. (2004) and Djankov et al. (2003) support our hypothesis that predation is affected by both institutional characteristics and factor accumulation. Our paper does not only analyze the effect of institutions on capital accumulation, as most of the literature does (see Acemoglu et al., 2005, for a complete survey), instead, it also studies the effect of capital accumulation on the institutional structural change.

We study the role of governance in building institutions by analyzing the effect of policies that aim to improve institutions. We analyze two type of policies to build institutions: *(i)* "costless" policies that do not drain resources from the economy but reduce the productivity of predation, like legal reforms that hinder predation without spending resources on it. *(ii)* policies that require resources from the economy in order to deter predation. Costless policies have a clear effect: when a zero-cost policy that hinders predation is implemented, incentives to predate decline. This generates an institutional structural change, labor moves from predation to production. The institutional structural change generates a sectorial institutional structural change that reinforces the institutional structural change by increasing the labor share and, consequently, the reward for working in production. The feedback between institutional and sectorial structural changes stimulate capital accumulation, and the economy converges to a new steady state with a higher per capita capital level.

Costly policies devoted to build institutions, however, have not results as clear as one would expect at the first glance. Consider that the government collects taxes and part of these taxes are devoted to hire workers that reduce the productivity of predation. A policy that intend to build institutions by raising the tax rate involves numerous offsetting mechanisms that make the policy result uncertain: *(i)* an increase in the tax rate increases the number of workers hired by the government to avoid predation; consequently, predation drops and labor supply increases. *(ii)* Since the government collects taxes from productive income and not from predation rents, income taxes are distortionary, and may promote predation. *(iii)* A higher tax rate enlarges the number of tax officers, reducing the supply of labor in production. *(iv)* A higher tax rate enlarges the "government sector", which is the most intensive sector in labor (it only uses labor). This increases the labor share in the economy, discouraging predation. *(v)* An increase in the tax rate has a negative direct effect on the after-tax return on savings, that may be, or not, compensated by the reduction on the fraction of income that goes to predation (effect *(i)*). Only when the tax rate is low enough the effect of tax rate has a conclusive result: the increase in the tax rate generates a positive effect on per capita capital. Otherwise such effect is uncertain.

There is a large amount of literature focused on studying the allocation of resources to productive and unproductive activities (see among others, Murphy et al., 1991, 1993, Acemoglu, 1995, Acemoglu and Verdier, 1998, Schrag and Scotchmer, 1993, Grossman and Kim, 1996, 2002, Tornell and Lane, 1999 and Chassang and Padró-i-Miquel 2009, 2010). The papers most related to ours are the ones that establish a connection between predation and the factorial composition of income, such as Dal Bó and Dal Bó (2011), Zuleta (2004) and Andonova and Zuleta (2009) and Bethencourt and Perera-Tallo (2015). However, no one of these contributions analyze the feedback process between sectorial and institutional structural changes, which is the focus of our paper. The reason is that either the model is static (Dal Bó and Dal Bó, 2011), or the labor share is considered constant (Zuleta, 2004, and Andonova and Zuleta, 2009) or there is just one sector and sectorial structural change is not possible (none of other papers have considered multi-sectorial models or sectorial structural change).

There exists also a considerable number of papers on sectorial structural change (see for example Restuccia, Yang and Zhu, 2008, Gollin, Parente and Rogerson, 2002, 2004, 2007, and Córdoba and Ripoll, 2009), however, no one of them deal with predation or institutional structural change.

This paper is organized as follows. Section 2 summarizes the empirical evidence on predation and development. Section 3 develops a model of three sectors: agriculture, manufacturing and predation. Section 4 analyzes agents' decisions and section 5 defines the equilibrium. Section 6 explains the relationship between sectorial and institutional structural changes. Section 7 presents the dynamic behavior of the economy. Section 8 analyzes how predation amplifies differences in productivity across countries. Section

9 analyzes the role of good governance in building institutions. Section 10 presents some extensions of the model. Section 11 concludes and the appendix presents proofs and technical details.

2 Empirical evidence on predation and development

Although many predatory activities are legal, the ones that are best measured are the illegal ones. This is the case, for instance, of economic crimes. In this respect, Bourguignon (1999) finds that the share of property crime in GDP is 1.5% for Latin America, while it is 0.5% for the United States. In a recent contribution Soares and Naritomi (2010) show that, for a wide sample of countries, regions with higher GDP per capita, such as North America and Western Europe, also display lower burglary and theft rates. Similarly, recent empirical evidence suggests that developing countries also show high levels of white-collar crimes. Those countries, typically characterized by large informal sectors, show high levels of both labor income tax evasion (Besley and Persson, 2014, and Alm, 2014) and capital income tax evasion (Cobham and Janský, 2018, and Crivelli et al., 2016). Moreover, since tax evasion is usually taking place in the shadow economy, it is often accompanied by other types of white-collar crimes. This accounts for the fact that developing countries also show higher levels of evasion of social security contributions and transfer fraud (Petersen et al. (2010)), as well as, money laundering (Hendriyetty and Grewal, 2017). Considering that corruption is defined as the abuse of public office for private gain then, a broad range of actions like bribery or embezzlement can be identified as pure acts of predation. In this respect, Treisman (2000), Paldam (2001, 2002), Brunetti and Weder (2003) and Rehman and Naveed (2007), among others, show that corruption is higher in developing countries. Moreover, countries which show high levels of corruption usually display high levels of other forms of predation. For instance, Morck et al. (2000) find that more corrupt countries also display more price manipulation.

Finally, regarding the empirical measures of predation derived from legal activities, the existence of measurement problems explains why there is not much research on it. One of the most studied cases is rent-seeking. The concept of rent-seeking involves activities that waste resources pursuing 'unearned' incomes. One of the first attempts to measure rent-seeking is Katz and Rosenberg (1989). They deal with the existence of pressure groups determining the composition of the public spending. They measure rent-seeking caused by government budgetary allocations as total change in the budget's proportional allocation for different purposes. For a sample of 20 developed and developing countries they find that poorer countries show high levels of rent-seeking. Monopolies are also included into the rent-seeking set of activities, since they generally imply the imposition of disadvantages on their competitors (Murphy et al., 1993). In this regard, Schwab and Werker (2018), using a rich dataset on manufacturing sectors for 100 countries and applying the methods from the competition-and-growth literature of Aghion and coauthors, find that developing countries shows higher levels of rent-seeking with higher markups. Moreover, there is abundant empirical evidence about the negative impact of rent-seeking on growth and development. For instance, Acemoglu (1995) finds that rent-seeking produces a drain of talent from the productive sector, whereas Baland and Francois (2000) find that rent-seeking destroys entrepreneurship.

3 The model

Time is continuous with an infinite horizon. There are two different goods in the economy: agricultural and manufactured goods, denoted by sub-indexes a and m respectively. Agricultural goods are used only for consumption, while manufactured goods are used for consumption and investment in physical capital:

$$Y_a(t) = C_a(t) \quad (1)$$

$$Y_m(t) = C_m(t) + \dot{K}(t) + \delta K(t) \quad (2)$$

where $Y_a(t)$ denotes the aggregate production in agriculture, $C_a(t)$ denotes the aggregate consumption in agriculture, $Y_m(t)$ denotes the aggregate production in manufacturing, $C_m(t)$ denotes the aggregate consumption in manufacturing, $K(t)$ denotes aggregate capital, $\delta \in (0, 1)$ denotes the depreciation rate and $\dot{K}(t) + \delta K(t)$ denotes the gross investment.

3.1 Technology

Production technologies of agricultural and manufactured goods are given by the following production functions:

$$Y_a(t) = \Gamma_a (K_a(t))^{\alpha} (Z_a(t))^{\beta} (L_a(t))^{1-\alpha-\beta} \quad (3)$$

$$Y_m(t) = \Gamma_m (K_m(t))^{\alpha} (L_m(t))^{1-\alpha} \quad (4)$$

where $\alpha \in (0,1)$, $\beta \in (0,1/2)$[3] and $\alpha + \beta < 1$; $K_a(t)$ and $K_m(t)$ denote, respectively, the physical capital used in agriculture and manufacturing, $L_a(t)$ and $L_m(t)$ denote, respectively, the amount of labor used in agriculture and manufacturing; and $Z_a(t)$ denotes the amount of land used in agriculture. In order to capture the fact that agriculture is more land intensive than manufacturing, we have used the extreme but simple assumption that only agriculture uses land. These technologies capture the empirical facts found by Valentinyi and Herrendorf (2008): the agriculture sector has a similar capital share as other sectors but it is more land-intensive. These two facts together imply that agriculture has a lower labor share than other sectors.

The per capita productions of the agricultural and manufacturing sectors are given by:

$$y_a = \Gamma_a k_a^{\alpha} z_a^{\beta} l_a^{1-\alpha-\beta} \quad (5)$$

$$y_m = \Gamma_m k_m^{\alpha} l_m^{1-\alpha} \quad (6)$$

where lower case letters indicate per capita terms.

3.2 Preferences

The economy is populated with many identical dynasties of homogeneous agents. To simplify, we assume that population is constant. Preferences of a dynasty are given by the following function:

$$\int_{\tau}^{\infty} \ln\left(c(t) - \overline{c}\right) e^{-\rho(t-\tau)} dt, \quad c(t) = \begin{cases} c_a(t) & \text{if } c_a(t) \leq \overline{c} \\ \overline{c} + c_m(t) & \text{if } c_a(t) \geq \overline{c} \end{cases}$$

where $c_a(t)$ and $c_m(t)$ denote, respectively, the per capita consumption of agricultural and manufactured goods in period t, and $\rho > 0$ is the discount

[3] The assumption that $\beta < 1/2$ is used in the proof of the existence and unicity of the steady state.

rate of the utility function. Thus, these preferences imply a "food problem": households do not consume manufactured goods until reaching a certain "subsistence" level of consumption of agricultural goods, denoted by $\overline{c}$. We will concentrate in the case in which the consumption of manufactured good is positive, which occurs when the capital is above a certain threshold level of capital, $k^{\min}$ defined in the appendix (see 45).

3.3 The predation technology

Each period, agents are endowed with fixed z units of land and one unit of time which can be devoted to two types of economic activities: to produce goods (agricultural or manufactured goods), l, and/or to commit predation, l_p, that is,

$$1 = l(t) + l_p(t) \tag{7}$$

We define predation as any activity which implies the use of resources to obtain incomes without generating production. As we explained in the introduction, predation includes property crimes, fraud, corruption, lobbying, rent-seeking, etc. The amount of income obtained through predation is denoted by $\widetilde{y}(t)g\left(l_p\right)$, where $\widetilde{y}(t)$ is the per capita (gross) production and $g : \Re_+ \rightarrow [0,1]$ is the fraction of per capita (gross) production that each agent predates, which depends positively on the amount of time devoted to such activity, l_p. We assume that the function $g(.)$ is strictly increasing, strictly concave, continuous and differentiable of second order and $g(0) = 0$, $g(1) < 1$ and $g'(0) \geq 1$.

4 Agents' decisions

We will concentrate on the case where the consumption is above subsistence level (the "food problem" is solved), and, therefore, there is consumption of manufactured good.

4.1 Households:

Household maximization problem is as follows:

$$\max_{\{c(t),l(t),l_p(t),b(t)\}_{t=0}^{\infty}} \int_0^{\infty} \ln(c(t)-\overline{c})e^{-\rho t}dt \tag{8}$$

$$s.t:$$

$$c(t) = c_m(t)+\overline{c} \tag{9}$$

$$\dot{b}(t) = \underbrace{w(t)l(t)+r(t)b(t)+w_z(t)z\text{-}g(\widetilde{l_p}(t))y(t)}_{\text{Net income from production}}+ \underbrace{g(l_p(t))\widetilde{y}(t)}_{\text{Predation income}}\text{-}c_m(t)\text{-}p_a(t)\overline{c}$$

$$l(t)+l_p(t) = 1$$

$$y(t) = w(t)l(t)+(\delta+r(t))b(t)+w_z(t)z$$

where $b(t)$ denotes the amount of assets of the household, $w(t)$ the wage per unit of labor, $r(t)$ the net return on assets, $w_z(t)$ the land renting price, $y(t)$ the household's gross income and $p_a(t)$ the price of agricultural goods in terms of manufactured goods. We normalize the price of manufactured goods to one. Since $r(t)$ is the net return on assets, $\delta + r(t)$ is the gross interest rate, which is the one that appears in the definition of gross income. The sign "$\sim$" over a variable means that such variable is a per capita variable of the economy and therefore, the household cannot decide on it. Thus, $\widetilde{l_p}$ denotes the per capita labor devoted to predation and $\widetilde{y}$ denotes the per capita gross income. (Net) income coming from the production sector is equal to labor income from the production sector $w(t)l(t)$, plus financial income $r(t)b(t)$, plus land rents $w_z(t)z$, minus the amount of this income that is predated by other agents in the economy $g(\widetilde{l_p}(t))y(t)$. The other source of income comes from the predation sector which is equal to $g(l_p(t))\widetilde{y}(t)$. It is straightforward from the definition of preferences that when an agent enjoys a consumption level above the subsistence consumption level, the agent is going to consume the subsistence level of agricultural goods $\overline{c}$. Thus, the total expenditure on consumption is equal to the expenditure on agricultural goods, $p_a(t)\overline{c}$, plus the expenditure on consumption of manufactured goods $c_m(t)$. The increase in the household's assets, $\dot{b}(t)$, is equal to its savings, which is equal to its income (the one from production plus the one from predation) minus the expenditure on consumption goods, $c_m(t)+p_a(t)\overline{c}$.

The first order conditions for the interior solution are as follows:

$$w(t)\left[1-g(\widetilde{l_p}(t))\right] = g'_{l_p}(l_p(t))\widetilde{y}(t) \tag{10}$$

$$\frac{\dot{c_m}(t)}{c_m(t)} = (r(t)+\delta)\left(1-g(\widetilde{l_p}(t))\right)-\delta-\rho \tag{11}$$

Equation (10) specifies that the net wage in the production sector after predation should be equal to the marginal payment of predatory activities. That is, the marginal payment of the time devoted to each activity should be alike. Equation (11) is the typical Euler equation: the speed at which consumption grows depends positively on the return on savings, $(r(t)+\delta)\left(1-g(\widetilde{l_p}(t))\right)-\delta$ and negatively on the discount rate of the household, ρ.

The following transversality condition should also be satisfied:

$$\lim_{t\to+\infty}\frac{1}{c_m(t)}e^{-\rho t}b(t)=0$$

4.2 Firms:

Firms maximize profits. The optimization problem of firms in agriculture in per capita terms is defined by:

$$\begin{array}{ll}\max\limits_{y_a,l_a,z_a,k_a} & p_a y_a - w l_a - w_z z_a - (\delta+r)k_a \\ s.t.: & \Gamma_a k_a^{\alpha} z_a^{\beta} l_a^{1-\alpha-\beta} \geq y_a\end{array} \tag{12}$$

while the optimization problem of firms in manufacturing is given by:

$$\begin{array}{ll}\max\limits_{y_m,l_m,k_m} & y_m - w l_m - (\delta+r)k_m \\ s.t.: & \Gamma_m k_m^{\alpha} l_m^{1-\alpha} \geq y_m\end{array} \tag{13}$$

The first order conditions of the above problems are:

$$(1-\alpha-\beta)p_a\Gamma_a\frac{k_a^{\alpha}{z_a}^{\beta}{l_a}^{1-\alpha-\beta}}{l_a} = w \tag{14}$$

$$\alpha p_a\Gamma_a\frac{k_a^{\alpha}{z_a}^{\beta}{l_a}^{1-\alpha-\beta}}{k_a} = (\delta+r) \tag{15}$$

$$\beta p_a\Gamma_a\frac{k_a^{\alpha}{z_a}^{\beta}{l_a}^{1-\alpha-\beta}}{z_a} = w_z \tag{16}$$

$$(1-\alpha)\Gamma_m\frac{k_m^{\alpha}{l_m}^{1-\alpha}}{l_m} = w \tag{17}$$

$$\alpha\Gamma_m\frac{k_m^{\alpha}{l_m}^{1-\alpha}}{k_m} = (\delta+r) \tag{18}$$

These standard conditions mean that firms hire a factor until reaching the point at which the marginal productivity of the factor is equal to its price.

5 Equilibrium definition

The definition of equilibrium is standard: equilibrium occurs when agents maximize their objective functions and markets clear. Steady state equilibrium is an equilibrium in which both the allocation and prices always remain constant over time.

Definition 1 *An equilibrium is an allocation* $\left\{c_m(t), c_a(t), l(t), l_p(t), b(t), y_a(t), z_a(t), l_a(t), k_a(t), y_m(t), l_m(t), k_m(t), \widetilde{l_p}(t), \widetilde{y}(t)\right\}_{t=0}^{\infty}$ *and a vector of prices* $\{p_a(t), w(t), r(t), w_z(t)\}_{t=0}^{\infty}$ *such that* $\forall t$ *the following conditions hold:*

- Households maximize their utility, that is, $\{c_m(t), l(t), l_p(t), b(t)\}_{t=0}^{\infty}$ is the solution of the household's maximization problem (8) and $c_a(t) = \overline{c}$.
- Firms maximize profits, that is, $\forall t$ $y_a(t)$, $l_a(t)$, $z_a(t)$, $k_a(t)$ and $y_m(t)$, $l_m(t)$, $k_m(t)$ are the solution of the firms' optimization problem (12) and (13).
- Capital market clears: $\forall t$ $k_a(t) + k_m(t) = k(t) = b(t)$.
- Labor market clears: $\forall t$ $l_a(t) + l_m(t) = l(t)$.

- Land market clears: $\forall t \; z_a(t) = z$.
- Good Market clears: $\overline{c} = y_a(t)$, $c_m(t) + \dot{b}(t) + \delta b(t) = y_m(t)$.
- Finally, since households are identical, per capita variables coincide with household variables: $\forall t \; \widetilde{l}_p(t) = l_p(t)$ and $\widetilde{y}(t) = w(t)l(t) + (\delta + r(t))b(t) + w_z(t)z$.

Definition 2 *Steady state equilibrium is an equilibrium in which both the allocation and prices always remain constant over time.*

6 Sectorial and institutional structural changes and per capita capital

6.1 Sectorial composition, labor share and incentives to predate

We define the labor share, λ, in the productive sector as the fraction of labor income over the aggregate production in the economy:

$$\lambda = \frac{wl}{y} = \frac{wl_a + wl_m}{p_a y_a + y_m}$$

Lemma 3 *Labor share is a decreasing function of the portion of productive labor devoted to agriculture, $\psi_a \equiv l_a/l$.*

The labor share in the economy decreases with the portion of productive labor that is devoted to agriculture. The reason is that agriculture has a lower labor share than manufacturing.

Lemma 4 *The portion of labor devoted to predation, l_p, is a strictly decreasing function of labor share.*

A higher labor share increases the relative reward for working with respect to predation, which encourages working in productive activities and discourages predation. We will denote by $l_p(\lambda)$ and $l(\lambda) = 1 - l_p(\lambda)$, the labor devoted to predation and to production respectively, as functions of the labor share λ.

6.2 Labor allocation across sectors, activities and per capita capital

Proposition 5 *The portion of labor devoted to agriculture at equilibrium, ψ_a, and the portion of labor devoted to predation at equilibrium, l_p are strictly decreasing functions of k, Γ_a and z, and a strictly increasing functions of $\overline{c}$. The labor share and the portion of labor devoted to production at equilibrium, l, are strictly increasing functions of k, Γ_a and z, and strictly decreasing functions of $\overline{c}$.*

Households' preferences imply that households do not consume manufactured goods until reaching a certain "subsistence" level of consumption of agricultural goods, $\overline{c}$. When resources of the economy (per capita capital or land) expand or agricultural technology improves, the amount of labor required to produce the subsistence level of consumption goes down. Consequently, labor shifts from agriculture to manufacturing. This "sectorial structural change" increases the labor share, discouraging predation and fostering work in productive activities. That is, generating a "institutional structural change". Exactly the opposite effects occur if the subsistence level of consumption rises.

From now on, we will denote by $\lambda(k)$ the (increasing) function that relates the labor share with the amount of per capita capital. We will simplify notation as follows: $l_p(k) = l_p(\lambda(k))$ and $l(k) = l(\lambda(k))$, when this does not induce confusion.

7 Dynamic behavior: sectorial and institutional structural changes along the transition

The dynamic system that defines the dynamic behavior of the economy when the consumption of manufactured goods is positive (when $c_m > 0$) is as follows:

$$\dot{k}(t) = y_m(k(t)) - c_m(t) - \delta k(t) \tag{19}$$

$$\frac{\dot{c_m}(t)}{c_m(t)} = (r(k(t)) + \delta)[1 - g(l_p(k(t)))] - \delta - \rho \tag{20}$$

where $y_m(k(t))$ is the function that relates per capita production of the manufacturing sector to per capita capital at equilibrium. This function takes into account the fact that, at equilibrium, some resources of the economy are devoted to the production of agriculture and others to predation. $r(k(t))$ is the function that relates the interest rate at equilibrium to per capita capital. These two functions are defined in the appendix (in the subsection Dynamic System).

Proposition 6 *There exists $\Omega(\delta, \rho)$ such that if $\left(\frac{\overline{c}}{\Gamma_a z^\beta}\right)\left(\frac{1}{\Gamma_m}\right)^{\frac{\alpha}{1-\alpha}} < \Omega(\delta, \rho)$ then there exists an unique steady state equilibrium in which consumption of manufactures is positive, $c_m > 0$ (where $\Omega(\delta, \rho)$ is defined at the appendix).*

In order that there is a steady state in which per capita capital is large enough to make possible the consumption of manufactures, some conditions are required: first, the amount of resources devoted to produce the subsistence level of agriculture goods should not be too high; otherwise the consumption of manufactures is not possible, due to the existence of a "food problem". This means that the subsistence level of consumption of agricultural goods $\overline{c}$ should not be too high, and the productivity of agriculture Γ_a and the amount of land z should be large enough. Second, the capital accumulated at the steady state should be large enough in order that households are wealthy enough to be able to consume manufactures. Consequently, the productivity of the manufacturing sector Γ_m should be large enough, and the depreciation rate δ and the discount rate of the utility ρ should be low enough. This would imply that households save sufficiently to reach a high enough level of per capita capital at the steady state to allow the consumption of manufactures.

We will focus on the case that there is a steady state with positive consumption of manufactures. Thus, we assume from now on that $\left(\frac{\overline{c}}{\Gamma_a z^\beta}\right)\left(\frac{1}{\Gamma_m}\right)^{\frac{\alpha}{1-\alpha}} < \Omega(\delta, \rho)$.

The phase diagram in Figure 2 shows that the dynamic behavior of the economy is characterized by a typical saddle point dynamic[4]. This means that, given the initial level of per capita capital, there is a unique equilibrium path, which converges to the steady state. When the initial amount of per capita capital is lower than the steady state level, proposition 5 implies that both sectorial and institutional structural changes occur in the economy. The lower part of figure 2 displays these twofold structural sectorial change. The consumption of manufactured goods grows generating a

[4]See appendix for technical details.

sectorial structural change in which labor shifts from agriculture to manufacturing. As labor shifts to manufacturing, the labor share increases. Since the reward for working increases with the labor share, this promotes an institutional structural change in which labor devoted to predation shifts to production. This increases production and promotes further the sectorial structural change. Thus, the feedback process between institutional and sectorial structural changes occurs along the transition to the steady state.

The sectorial and institutional structural changes that the model predicts are consistent with the empirical literature, which finds that: first, the percentage of workers in agriculture in developing countries is much higher than the one in developed ones, and, second, the size of predation sector is relatively large in developing countries.

8 The amplification effect of sectorial and institutional structural changes

8.1 The effect of an improvement in the technology of manufactures

Phase diagram in Figure 3 displays the dynamic effect of an improvement in the technology of the manufacturing sector, Γ_m. When there is an improvement in the technology of the manufacturing sector, the production and the marginal productivity of capital in this sector rise. As a consequence, the $\dot{k}$ locus goes up and the $\dot{c}$ locus moves to the right. The economy goes towards a new steady state with a higher level of capital, a lower portion of labor devoted to predation and a higher portion of labor and capital devoted to the manufacturing sector, as it is displayed in lower part of figure 3. Throughout the transition there is also an *amplification effect* due to the feedback between the sectorial and the institutional structural change: when per capita capital goes up, a sectorial structural change emerges, that is, labor shifts from agriculture to manufacturing. This sectorial structural change increases the reward for working since the labor share rises, and this generates an institutional structural change in which labor shifts from predation to production. This institutional structural change amplifies the effect of technological change on capital accumulation: since institutional structural change has a direct positive effect on the portion of the return on capital

Figure 2: Sectorial and institutional structural changes along transition

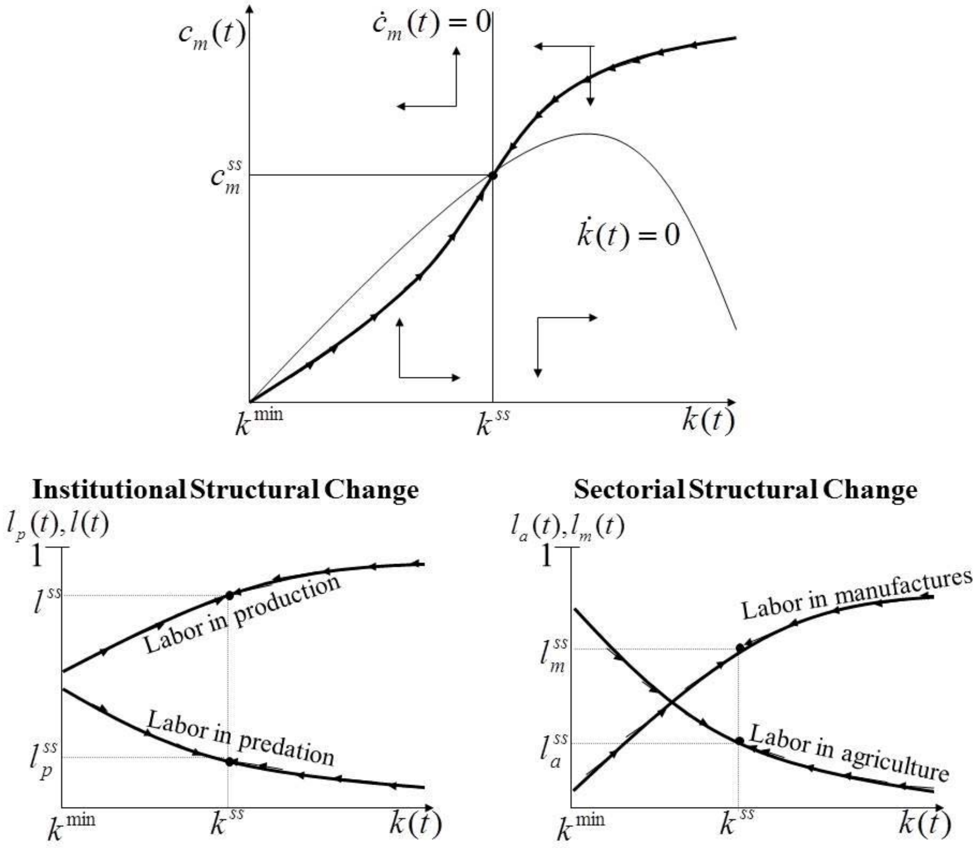
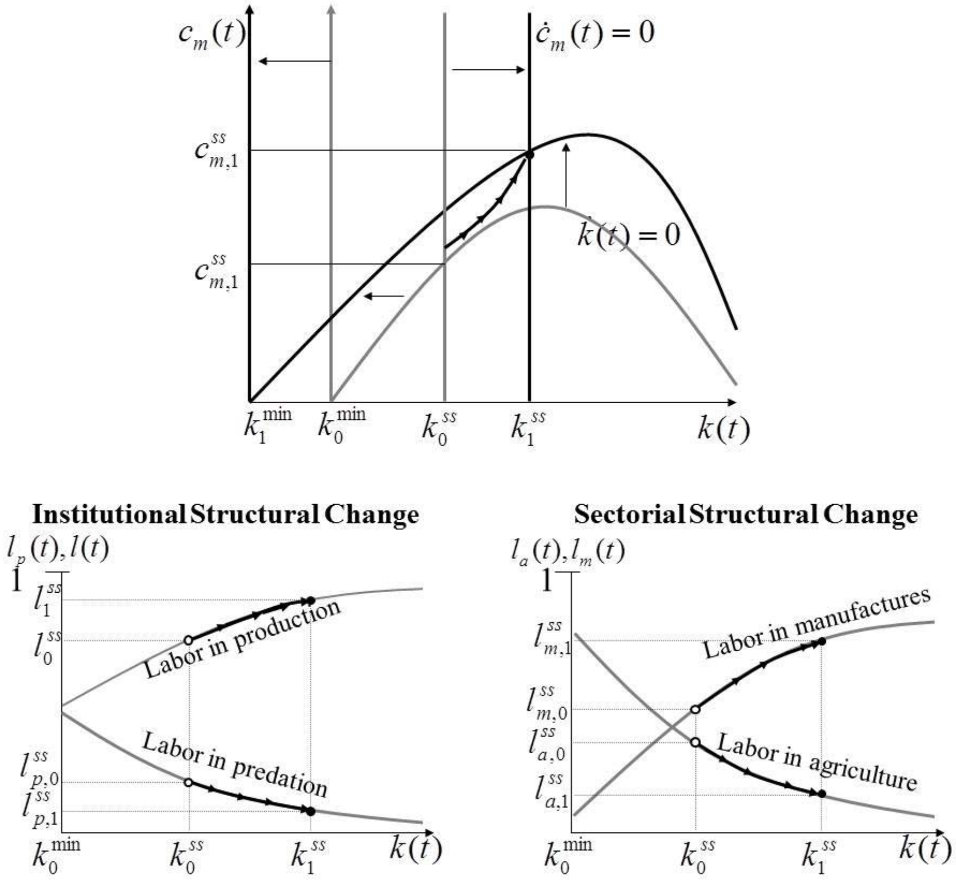


Figure 3: The effect of technological change in manufacturing



that goes to savers, this produces additional capital accumulation and so, additional sectorial structural change.

8.2 The effect of an improvement in the technology of agriculture

Proposition 7 *Both the labor devoted to predation at the steady state,* l_p^{ss}*, and the portion of labor devoted to agriculture at the steady state,* $\frac{l_a^{ss}}{l^{ss}}$*, decrease with the technological level of agriculture* Γ_a*.*

The above proposition says that and improvement in the technology of agriculture generates both sectorial and institutional structural change. An interesting feature of proposition 7 is that when the agriculture technology improves, this increase in productivity does not only affect agriculture, instead, it also spreads to manufacturing. The reason is that the improvement of technology in the agriculture allows to produce the subsistence level of consumption $\overline{c}$ using less resources. Thus, a technological change in agriculture releases resources from agriculture, generating a sectorial structural change in which resources shifts from agriculture to manufacturing. This sectorial structural change increases the labor share and the reward for working, reducing predation and generating an institutional structural change in which labor shifts from predation to production. This institutional structural change generates an additional increase in the production of manufacturing, besides the direct reallocation of resources from agriculture to manufacturing. Thus, the technological change in agriculture spreads to the whole economy and the institutional structural change amplifies the effect of such technological change.

9 The role of good governance in building institutions

In this section we extend the benchmark model in order to analyze the important role that good governance plays in building institutions and deterring predation. We modify the model in order to incorporate two types of policies that government may implement to reduce predation and build better institutions: *(i)* "zero-cost" policies, i.e., policies that do not require resources like, for instance, legal changes that may improve institutions without using many resources. *(ii)* Costly policies, i.e., the ones that require resources in order to be implemented. Probably most measures that governments can implement to deter predation and to build better institutions are costly. Therefore, in this section we consider the possibility that institutions are costly and that building institutions requires draining resources from the economy. We will see that costly policies generate complex offsetting mechanisms and different types of feedback processes. These offsetting mechanisms and feedback processes make the effect of those policies non-trivial and not as clear as one would expect at the first glance.

In this section we modify the model in order to incorporate the two types of policies explained above: costless policies and policies which require resources to be implemented. More precisely, we modify the predation technology of the benchmark model in two respects: *(i)* In order to capture zero-cost policies, we introduce parameter $\xi \in \left[\underline{\xi}, \overline{\xi}\right]$ as a measure of the institutional quality. An increase in the institutional quality reduces the productivity of predation. In this context, a costless policy to build institutions (an increase in parameter ξ) would consist in, for instance, legal changes which improve institutions with reduced costs. *(ii)* In order to introduce costly policies, we will consider that the government hire l_g workers (in per capita terms) in order to avoid predation. Thus, the amount of income obtained through predation results to be equal to $g\left(l_p\right)\varphi\left(l_g,,\xi\right)\widetilde{y}(t)$, with $g(.)$ having the same properties defined in the benchmark model. Function $\varphi\left(l_g,\xi\right)$ is a strictly decreasing (in both arguments) continuous function in $[0,1]\times\left[\underline{\xi},\overline{\xi}\right]$ and twice differentiable function in $(0,1]\times\left[\underline{\xi},\overline{\xi}\right]$ and such that $\varphi\left(0,\xi\right)=1$, $\lim_{l_g\to 0}\varphi'_{l_g}\left(l_g,\xi\right) = -\infty$, $\lim_{l_g\to 0}\left[\varphi'_{l_g}\left(l_g,\xi\right)l_g\right] = 0$. An example of such function would be $e^{-\xi l_g^{\mu}}$, with $\mu\in(0,1)$. Government expenditures are financed with income taxes with tax rate τ. Income taxes levy on productive incomes but do not affect "predation rents". Furthermore, we also assume that households receive transfers from the government. Thus, the household' maximization problem would be as follows:

$$\max_{\{c(t),l(t),l_p(t),b(t)\}_{t=0}^{\infty}} \int_0^{\infty} \ln(c(t)-\overline{c})e^{-\rho t}dt$$

$$\begin{aligned}
&s.t: \\
c(t) &= c_m(t)+\overline{c} \\
\dot{b}(t) &= \underbrace{w(t)\,l(t)+r(t)b(t)+w_z(t)z\text{-}g(\widetilde{l}_p(t))\varphi\left(\widetilde{l}_g(t),\xi\right)y(t)\text{-}\tau y(t)+}_{\text{After tax net income from production}} \\
&\underbrace{g(l_p(t))\varphi\left(\widetilde{l}_g(t),\xi\right)\widetilde{y}(t)}_{\text{Predation income}}\text{-}c_m(t)\text{-}p_a(t)\overline{c}+tr(t) \\
l(t)+l_p(t) &= 1 \\
y(t) &= w(t)l(t)+(\delta+r(t))b(t)+w_z(t)z
\end{aligned}$$

The first order condition with respect to l_p is as follows:

$$w(t)\left[1-\tau-g(\widetilde{l_p}(t))\varphi\left(l_g(t),\xi\right)\right] = g'_{l_p}(l_p(t))\varphi\left(l_g(t),\xi\right)\widetilde{y}(t) \qquad (21)$$

Note that taxes are distortionary. Since taxes levy on only productive incomes, an increase in the tax rate reduces the reward for working in productive activities and, consequently, promotes predation. However, a portion of tax collection is devoted to hire government employees in order to avoid predation, reducing the reward for predation (reducing "$\varphi(.)$"). Consequently, the net effect of taxes on predation will result uncertain.

The budget constraint of the government is as follows:

$$\vartheta\tau y = wl_g; \quad (1-\vartheta)\tau y = tr \qquad (22)$$

where the portion ϑ of tax collection is devoted to avoid predation and the portion $(1-\vartheta)$ to provide transfers. We will call the portion ϑ, government commitment, since it represents the effort that the government does in order to build institutions targeted to deter predation.

Now productive labor is devoted to three activities: agriculture, manufacturing and government. Thus, the labor market clearing condition is as follows:

$$l = l_a + l_m + l_g \qquad (23)$$

Proposition 8 *Assume that* $\frac{g'\left(\frac{1}{2}\right)\frac{1}{2}}{1-g\left(\frac{1}{2}\right)} > 1-\alpha-\beta$ *and that* $\min_{l_p\in\left[1.\frac{1}{2}\right]} -\varepsilon_{l_p}^{g'(l_p)}(l_p) > 1-\alpha$*, where* $\varepsilon_{l_p}^{g'(l_p)}(l_p) = \frac{g"(l_p)l_p}{g'(l_p)}$ *is the elasticity of* $g'(l_p)$*. There exists* $\overline{\tau}\in(0,1)$ *such that if* $\tau\in[0,\overline{\tau}]$*, then there exists a unique steady state. The capital at the steady state is a strictly increasing function of the institutional quality* ξ*. Furthermore:*

- *there exists* $\tau_\vartheta \in (0,\overline{\tau}]$ *such that if* $\tau\in(0,\tau_\vartheta)$*, then the capital at the steady state is a strictly increasing function of the government commitment,* ϑ*.*

- *There exists* $\widehat{\tau}\in(0,\overline{\tau}]$ *such that if* $\tau\in(0,\widehat{\tau})$*, then the capital at the steady state is a strictly increasing function of the tax rate,* τ*.*

The above proposition implies that there is only one type of policies to build institutions with a clear effect: those that do not involve drain of resources from production, that is, costless policies (increases in ξ). These policies reduce incentives to predate, generating institutional structural change which promotes capital accumulation and sectorial structural change. This, in turn, fosters further the institutional structural change, generating a transition to a new steady state in which per capita capital is higher than the one at the initial steady state.

Costly policies (policies that drain resources from productive activities) have uncertain effects due to the complex offsetting mechanisms that involve. An increase in the government commitment, ϑ, reduces predation but extracts labor from the production of goods. Thus, the increase in government commitment does not always have a positive effect on per capita capital. When the tax rate is low, there are not many public officers avoiding predation, which implies that they are very productive. Thus, in this case, an increase in government commitment involves a recruitment of more public officers with high marginal productivity in deterring predation. The reduction in predation that these public officers generates, overcomes the negative effect of detracting resources from the production of goods. When the tax rate is high, the opposite may occurs.

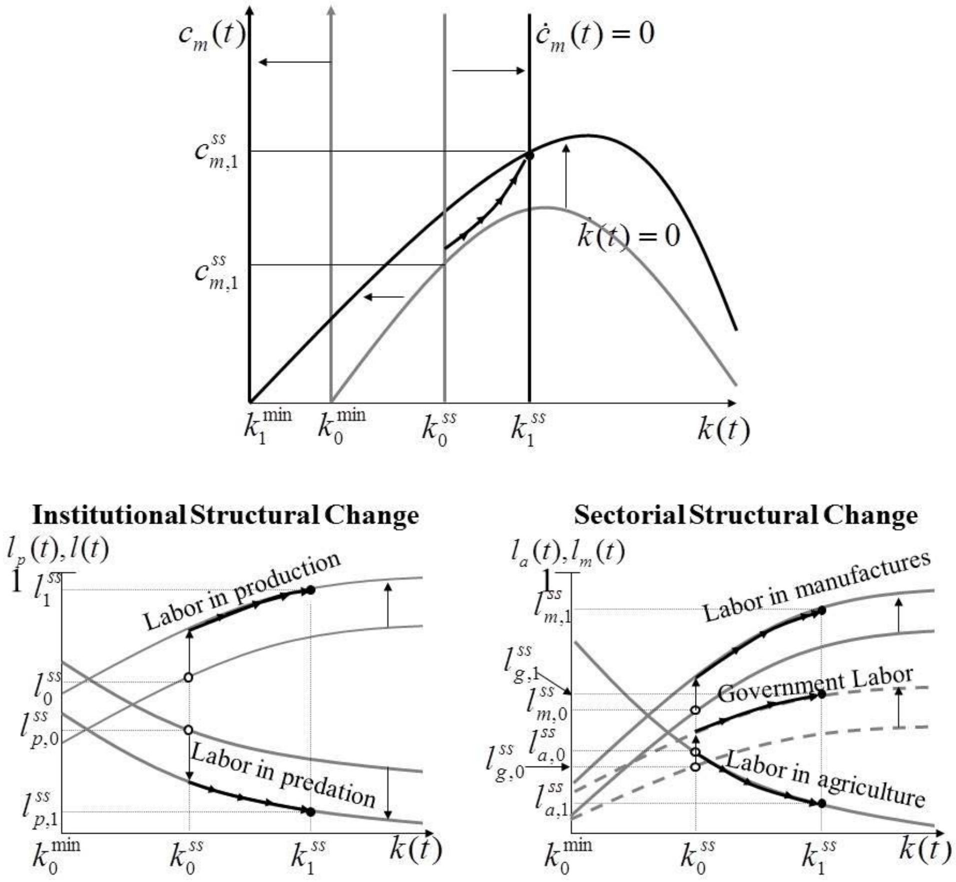
An increase in the tax rate involves numerous complex and offsetting mechanisms: *(i)* an increase in the tax rate expands the number of workers hired by the government to avoid predation; consequently, predation drops and labor supply increases. *(ii)* Since the government collects taxes from productive income and not from predation rents, income taxes are distortionary, and may promote predation. *(iii)* A higher tax rate enlarges the number of workers hired by the government, reducing the supply of labor in production. *(iv)* A higher tax rate enlarges the "government sector", which is the most intensive sector in labor (it only uses labor). As a consequence, the labor share increases, discouraging predation. *(v)* An increase in the tax rate has a negative direct effect on the after-tax return on savings, that may be, or not, compensated by the reduction on the fraction of income that goes to predation (see *(i)*). Thus, given all these offsetting mechanisms, is not surprising that the relationship between the tax rate and the per capita capital, at the steady state, is not monotonic. We prove that when the tax rate is low enough, an increase in the tax rate has a positive net effect. This is due to the fact that when tax rate is low, public officers have a high marginal productivity in reducing predation. Consequently, positive effects of increasing taxes overcome negative effects.

Note that in addition to the feedback process between sectorial and institutional structural changes of the benchmark model, there are also two other feedback processes directly related with taxes: *(i)* if predation drops, labor devoted to production increases, generating more tax revenues. This allows to the government to hire more public officers who reduce predation further. *(ii)* A reduction in predation and an increase in labor devoted to production increase tax revenues, generating a reallocation of labor in favor of the government sector. Since the government sector has the highest labor share, this enlarges the labor share of the economy, promoting working in productive sectors and discouraging predation. These additional feedback mechanisms make harder to analyze the role of the good governance in buildings institutions.

Figure 4 displays the effect of an increase in the tax rate when the tax rate is low enough. In this case, an increase in the tax rate reduces the productivity of predation[5], encouraging individuals to devote more resources to productive sectors. Since, the economy has reached the minimum amount of consumption in agriculture; workers that were allocated to predation are now flowing to manufacturing generating a sectorial structural change. Furthermore, since there are more resources devoted to production, the level of tax revenue is higher and so, there are more government workers deterring predation. Graphically, the reallocation of workers at the moment of change in the tax rate τ implies a jump down in the predation curve, $l_p(k)$, and a jump up in the productive labor curve, $l(k)$, in the labor in manufacturing, $l_m(k)$, and the labor hired by the government, $l_g(k)$. This is displayed in the lower part of Figure 4. As a consequence, the production and the marginal productivity of capital in manufacturing rise, implying that the $\dot{k}$ locus goes up and the $\dot{c}$ locus moves to the right. The economy goes towards a new steady state with a higher level of capital, a lower portion of labor devoted to predation and a higher portion of labor and capital devoted to the manufacturing sector. An *amplification effect* appears along transition: when per capita capital goes up, a sectorial structural change emerges. The portion of

[5]In the case that the tax rate is low enough, an increase of the tax rate implies an increase in the number of government's workers with high marginal productivity in deterring predation. Thus, the reduction in the fraction of income that goes to predation is larger than the negative direct effect of reducing the disposable productive income. As a result, the "after tax and after predation" return on savings will go up, which will promote capital accumulation.

Figure 4: The effect of an increase in the tax rate (when the tax rate is low)



labor that goes to agriculture goes down, increasing the labor share, which generates an institutional structural change: labor shifts from predation to production. This institutional structural change involves more tax collection and more workers in the government sector that deter predation and increase the labor share, reinforcing the institutional structural change. Thus, the institutional structural change amplifies the effect of the tax rate on capital accumulation. Finally, the fall in predation, which has a direct positive effect on the portion of the return on capital that goes to savers, generates an additional incentive to expand the capital accumulation further.

Notice that, also when the tax rate is small enough, the effect of an increase in the institutional quality, ξ (that is, a zero-cost policy) and the effect of an increase in the government commitment, ϑ (that is, a costly policy) are both similar to the effect of an increase in the tax rate, described in Figure 4.

10 Some extensions

10.1 Specialization between "predators" and workers

In this section we will consider that labor is indivisible. Thus, agents should devote their entire unit of time either to work in productive activities (production) or to predate. Agents that work in production will be called workers, and agents that devote their time to predation will be called predators. We consider that agents are indexed by i in an interval $[0,1]$ and that they are uniformly distributed. We define $\eta(i)$ as the number of labor efficiency units of agent $i \in [0,1]$ when she devotes her unit of time to production, where ${}_0^1\eta(i)di = 1$. If she devotes her unit of time to predation, she gets as a reward for predation the following "rent":

$$\frac{\theta(i)}{l_p} g\left(l_p\right) y$$

where $\theta(i)$ is the "productivity" of agent $i \in [0,1]$ when she predates, and $l_p \equiv_0^{i_p} \theta(i)di$ is the (total) number of labor efficiency units devoted to predation. The portion of income that is predated, $g\left(l_p\right)$, is an increasing function in the "aggregate" predation effort, l_p. The amount of income that goes to predation, $g\left(l_p\right) y$, is distributed among predators according with their "productivity" in predation.

We assume that $\theta(i)/\eta(i)$ is decreasing, that is, agents with higher index i have comparative advantage in working, rather than in predation. This assumption implies that agents that devote their time to predation belong to a certain interval $[0, i_p]$, while agents that devote their time to production belong to the interval $[i_p, 1]$. The marginal agent i_p is the agent that is indifferent between devoting her unit of time to production or to predation.

We will consider a representative household in which their household members completely specialize in either production or in predation. That is, some household members devote their entire unit of time to work in the productive sector and other members devote their entire unit of time to predate. The representative household problem in this model would be as follows:

$$\max_{\{c(t),i_p(t),b(t)\}_{t=0}^{\infty}} \int_0^{\infty} \ln(c(t)-\overline{c})e^{-\rho t}dt$$

$$s.t:$$

$$c(t) = c_m(t)+\overline{c}$$

$$\dot{b}(t) = \underbrace{w(t)_{i_p(t)}^{1}\eta(i)di + r(t)b(t) + w_z(t)z\text{-}g\left(\widetilde{l}_p\right)y(t)}_{\text{Net income from production}} +$$

$$\underbrace{+_0^{i_p(t)}\frac{\theta(i)}{\widetilde{l}_p}di\ g\left(\widetilde{l}_p\right)\widetilde{y}(t)}_{\text{Predation income}} - c_m(t) - p_a(t)\overline{c}$$

$$y(t) = w(t)_{i_p(t)}^{1}\eta(i)di + (\delta + r(t))b(t) + w_z(t)z$$

The first order condition with respect to the marginal type, $i_p(t)$, is as follows:

$$w(t)\eta(i_p(t))\left(1-g\left(\widetilde{l}_p(t)\right)\right) = \frac{\theta(i_p(t))}{\widetilde{l}_p(t)}g\left(\widetilde{l}_p(t)\right)\widetilde{y}(t) \tag{24}$$

This condition means that the "marginal" agent that is indifferent between devoting her time to production or to predation, is the one that would get the same reward in any of these two possible activities. If this agent devotes her time to production, her reward is the wage in the productive sector minus the portion that is predated $w(t)\eta(i_p(t))\left(1-g\left(\widetilde{l}_p(t)\right)\right)$. If this agent devotes her time to predation, she gets as a reward $\frac{\theta(i_p(t))}{\widetilde{l}_p(t)}g\left(\widetilde{l}_p(t)\right)\widetilde{y}(t)$. Thus, this "marginal" agent is indifferent between devoting her unit of time to either production or predatory activities. Agents with higher types than the marginal agent, $i > i_p$, have comparative advantage in working in production, and they will devote their time to such activity, since their reward from working is higher than from predation. On the opposite, agents with lower types, $i > i_p$, will devote their time to predation. Note that it follows from the assumption of uniform distribution of i, that i_p also represents the portion of agents that devote their time to predation.

Lemma 9 *The portion of agents that devote their time to predation, i_p and the amount of efficiency units of labor devoted to predation $l_p \equiv_0^{i_p} \theta(i)di$ are* *both strictly decreasing functions of labor share. The portion of agents that devote their time to work,* $1 - i_p$*, and the amount of efficiency units of labor devoted to work* $l \equiv_{i_p}^{1} \eta(i)di$ *are both strictly increasing functions of labor share.*

It follows from this lemma that, in this extension, all the results of the benchmark model that we analyzed in previous sections go through. Thus, the fact that agents may specialize in production or in predation does not alter at all any of the results of the paper.

10.2 The role of human capital

In this section we analyze the role of human capital in reducing predation and so, in promoting institutional structural change. We now consider that each agent in the economy has h labor efficiency units if she works in production and one labor efficiency unit if she predates. We interpret h as the per capita amount of human capital. We assume that human capital affects the productivity in the production sector but not in predation. Thus, the representative household problem is as follows:

$$\max_{\{c(t),l(t),l_p(t),b(t)\}_{t=0}^{\infty}} \int_0^{\infty} \ln(c(t) - \overline{c})e^{-\rho t}dt$$

$$s.t.:$$

$$c(t) = c_m(t) + \overline{c}$$

$$\dot{b}(t) = \underbrace{w(t)h\,l(t) + r(t)b(t) + w_z(t)z - g(\widetilde{l_p}(t))y(t)}_{\text{Net income from production}} + \underbrace{g(l_p(t))\widetilde{y}(t)}_{\text{Predation income}} - c_m(t) - p_a(t)\overline{c}$$

$$l(t) + l_p(t) = 1$$

$$y(t) = w(t)l(t) + (\delta + r(t))b(t) + w_z(t)z$$

The first order condition with respect to predation is defined by:

$$w(t)h\left[1 - g(\widetilde{l_p}(t))\right] = g'_{l_p}(l_p(t))\widetilde{y}(t) \tag{25}$$

It follows from the above first order condition that human capital increases the reward for working in production and, consequently, increases the incentive to devote time to production, discouraging predation.

In this model we obtain the following results:

Lemma 10 *Labor share is a decreasing function of the portion of productive labor devoted to agriculture,* $\psi_a \equiv l_a/l$.

Lemma 11 *The portion of labor devoted to predation,* l_p*, is a strictly decreasing function of labor share.*

As in the benchmark model, a higher labor share increases the relative reward for working with respect to predation, which encourages the participation in production and discourages predation. Moreover, as in the benchmark model, the higher the portion of productive labor devoted to agriculture, the lower the labor share and, consequently, the higher the predation is.

Proposition 12 *The portion of labor devoted to agriculture at equilibrium,* ψ_a*, and the portion of labor devoted to predation at equilibrium,* l_p *are strictly decreasing functions of* h*. The portion of labor devoted to production at equilibrium,* l*, is a strictly increasing function of* h*.*

Human capital increases productivity, releasing resources from agriculture to manufacturing. This reallocation of resources implies an increase in the labor share, which reduces the incentive to predate. Furthermore, human capital has a direct effect in increasing the reward for devoting time to production, as it is reflected in first order condition (25). Thus, human capital plays an important role in reducing predation and accelerating sectorial and institutional structural changes.

10.3 Mixed Activities

In this section we explore the possibility of predation may generate some productive services. The reason behind is that some predatory activities might be identified as a service (producing services) and, therefore they might have a "productive" part. For instance, many economists hold that corruption, in some extent, may be considered a service to reduce the amount of bureaucracy (red tape). This is related with the observation that in many poor countries governments tolerate corruption to allow public officers to integrate their low levels of wages. There are more examples of these mixed activities which are in part predation, but that also may increase production (for instance, lobbying when it implies resources reallocation from unproductive to productive activities, rent-seeking from monopolies when rents are used to increase investment, etc.). In order to include these mixed activities, we incorporate the labor devoted to predation in the production of goods:

$$y_a = \Gamma_a k_a^\alpha z_a^\beta (l_a + \varepsilon l_{pa})^{1-\alpha-\beta} \tag{26}$$

$$y_m = \Gamma_m k_m^\alpha (l_m + \varepsilon l_{pm})^{1-\alpha} \tag{27}$$

$$l_p = l_{pa} + l_{pm} = 1 - l \tag{28}$$

where l_{pa} is the part of the predatory time (in per capita terms) that is assigned to produce agricultural goods and $l_{pm}(t)$ is the part of predatory time that is devoted to produce manufacturing goods; and $\varepsilon \in (0,1)$ is a parameter that indicates the labor efficiency units embodied in one unit of time devoted to predation. It follows from the firms maximization problem that the salary that predation time get in the productive sector is equal to εw.

Note that, even though part of the time devoted to predation is productive, it is still inefficient to predate, since it has a lower productivity than working. We define $\psi_a \equiv (l_a + \varepsilon l_{pa}) / (l + \varepsilon l_p)$ as the portion of productive labor devoted to agriculture (including the productive part of predation). Using this definition and the restriction that the per capita amount of labor is one, $l_p + l = 1$, it is straightforward to show that production in both productive sectors decreases with the amount of time devoted to predation:

$$y_a = \Gamma_a k_a^\alpha z_a^\beta (\psi_a (1 - l_p + \varepsilon l_p))^{1-\alpha-\beta} \implies \tag{29}$$

$$\frac{\partial y_a}{\partial l_p} = -(1-\alpha-\beta)\frac{\Gamma_a k_a^\alpha z_a^\beta}{l_a^{\alpha+\beta}} \psi_a^{1-\alpha-\beta}(1-\varepsilon) < 0 \tag{30}$$

$$y_m = \Gamma_m k_m^\alpha ((1-\psi_a)(1 - l_p + \varepsilon l_p))^{1-\alpha} \implies \tag{31}$$

$$\frac{\partial y_a}{\partial l_p} = -(1-\alpha)\frac{\Gamma_m k_m^\alpha}{l_m^\alpha}(1-\psi_a)^{1-\alpha-\beta}(1-\varepsilon) < 0 \tag{32}$$

Thus, the fact that part of the time devoted to predation is productive do not change anything: predation reduces production and devoting time to predation is always inefficient.

The household maximization problem is now as follows:

$$\max_{\{c(t),l(t),l_p(t),b(t)\}_{t=0}^{\infty}} \int_0^{\infty} \ln(c(t)-\overline{c})e^{-\rho t}dt$$

$$\begin{aligned}
&s.t:\\
&c(t) = c_m(t)+\overline{c}\\
&\dot{b}(t) = \underbrace{w(t)\,l(t)+w(t)\,\varepsilon l_p(t)+r(t)b(t)+w_z(t)z\text{-}g(\widetilde{l}_p(t))y(t)}_{\text{Net income from production}}+\\
&+\underbrace{g(l_p(t))\widetilde{y}(t)}_{\text{Predation income}}\text{-}c_m(t)\text{-}p_a(t)\overline{c}\\
&l(t)+l_p(t) = 1\\
&y(t) = w(t)l(t)+(\delta+r(t))b(t)+w_z(t)z
\end{aligned}$$

The first order condition with respect to predation is as follows:

$$w(t)\left[1-g(\widetilde{l}_p(t))\right] = g'_{l_p}(l_p(t))\widetilde{y}(t)+w(t)\varepsilon\left[1-g(\widetilde{l}_p(t))\right] \tag{33}$$

This condition means that the marginal reward of one unit of time devoted to work, $w(t)\left[1-g(\widetilde{l}_p(t))\right]$, should be equal to the marginal reward of one unit of time devoted to predation, which includes the rent from pure predation, $g'_{l_p}(l_p(t))\widetilde{y}(t)$, plus the rent from the "productive" part of predation $w(t)\varepsilon\left[1-g(\widetilde{l}_p(t))\right]$.

Lemma 13 *The portion of labor devoted to predation, l_p, is a strictly decreasing function of labor share.*

This lemma implies that all the results of the benchmark model that we analyzed in previous sections go through. Thus, the fact that some predatory activities may be partially productive does not alter at all the results of the paper.

11 Conclusions

This paper presents a neoclassical growth model with three sectors: agriculture, manufacturing and predation. The paper focuses on the interaction between sectorial structural change, reallocation of resources across sectors, and institutional structural change, reallocation of resources from unproductive activities (predation) to productive ones. In the model, households should first satisfy their subsistence needs of agricultural goods before starting to consume manufactured goods; the typical "food problem". As the country accumulates capital and subsistence needs begin to be satisfied, a sectorial structural change occurs: labor is reallocated from agriculture to manufacturing, which implies a higher weight of manufacturing in the added value of the economy. Due to the fact that agriculture is less labor-intensive than manufacturing, the sectorial structural change implies that (aggregate) labor share rises during the transition when the initial per capita capital is lower than the steady state level. This increase in the labor share implies a reduction in incentives to predate, and generates an institutional structural change, a reallocation of labor from predation to production. The institutional structural change foster further capital accumulation and sectorial structural change. Thus, this paper analyzes how sectorial structural change interacts with institutional structural change and capital accumulation, and the resulting feedback process.

This paper also contributes to better understanding of differences in per capita income and sectorial composition among countries. Despite many authors have identified differences in productivity as one of the main factors accounting for differences in per capita income, these differences in productivity are not empirically high enough to generate the differences that are observed in per capita income. Our paper offers an explanation that helps to reconcile theory with empirical literature: since changes in productivity of either agriculture or manufacturing generate sectorial and institutional structural changes, total effects of productivity result being bigger than direct effects. This feature of the model contributes to understand what Hall and Jones (1999) identify as the key factor to explain differences in income across countries: what they call "social infrastructure". The feedback between sectorial and institutional structural changes involves an amplification mechanism that generates greater differences in per capita income than the standard neoclassical one sector model does. What is important is that the existence of predation implies that the increase in the productivity of one particular sector does not only affect the productivity of this sector but also has a positive effect that spreads to the rest of the economy. For instance, the improvement in the productivity of agriculture reduces the amount of resources needed to reach the subsistence level of consumption and consequently involves a sectorial structural change in which labor shifts from agriculture to manufacturing. The spread arises due to the fact that this reallocation also affects the incentive to predate. More precisely, when labor is reallocated to the manufacturing sector, labor share increases, improving the reward for working and discouraging predation. This implies an institutional structural change in which labor shifts from predation to production, spreading the improvement in the agricultural sector to the whole economy, and in particular to the manufacturing sector and to the productive sector, building "social infrastructure". A technological change in manufacturing is also amplified by the feedback process between sectorial and institutional structural changes. An increase in the technology in manufacturing increases the return on capital, promoting capital accumulation, which generates a sectorial structural change in which labor shifts from agriculture to manufacturing. This sectorial structural change increases the labor share and the reward for working, generating and institutional structural change in which labor shifts from predation to production. This increases further the return on savings, promoting capital accumulation that generates again sectorial and institutional structural changes. Thus, the feedback process between the capital accumulation and sectorial and institutional structural changes amplifies the effect of any technological improvement. In other words, an improvement in the technology of production generates an improvement of the "social infrastructure" which affects to the whole economy.

We also analyzed the role that good governance plays in building institutions. We considered two types of policies at this respect: *(i)* costless policies, i.e., policies that reduces the productivity of predation without draining resources from the economy, like some legal reform; *(ii)* policies that require to be financed with taxes in order to reduce the productivity of predation. While zero-cost policies have a quite clear effect on the economy, policies financed with taxes have more complex and uncertain effects that one would expect at the first glance. First, when a costless policy is implemented, the productivity of predation drops, generating an institutional structural change in which labor shifts from predation to production. This increase in productive labor involves a sectorial structural change in which labor shifts from agriculture to manufacturing. Institutional and sectorial structural changes reinforce each other along transition. Furthermore, institutional and sectorial structural changes interact with capital accumulation: institutional structural change increases the return on savings promoting capital accumulation; whereas capital accumulation promotes sectorial structural change which fosters institutional structural change. These feedback mechanisms make the economy to converge to a new steady state in which per capita capital is higher than in the initial steady state. Second, in the case of costly policies, we consider that the government collects income taxes to hire workers that deter predation (the productivity of predation is decreasing in the number of the tax officers). Taxes are distortionary since they are collected from production, not from predation rents. When the tax rate increases, several offsetting mechanisms arise that make the result of policy uncertain: *(i)* an increase in the tax rate rises the number of tax officers, reducing predation and increasing labor supply. *(ii)* Since the government collects taxes from productive income and not from predation rents, income taxes are distortionary, and may promote predation. *(iii)* A higher tax rate enlarges the number of tax officers, reducing the supply of labor in production. *(iv)* A higher tax rate enlarges the "government sector", which is the most intensive in labor (it only uses labor). Thus, a higher tax rate implies a higher labor share, discouraging predation. *(v)* An increase in the tax rate has a direct effect on the after-tax return on savings, that may be, or not, compensated by the reduction on the fraction of income that goes to predation (also generated by this increase in the tax rate). Only when the tax rate is low enough, the effect of tax rate has a conclusive result: the increase in the tax rate generates a positive effect on per capita capital. Otherwise such effect is uncertain. Thus, the process of building institutions is much more complex and uncertain that may look like at the first glance. Given the results of the paper, it is hardly surprising that many countries have straggled in building institutions and "social infrastructure".

12 References

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13 Appendix

Proof of Lemma 3

Using equations (5), (6), (14) and(17), and defining $\psi_a \equiv \frac{l_a}{l}$,it follows that:

$$\lambda = \frac{wl}{y} = \frac{wl}{p_a y_a + y_m} = \frac{wl}{\frac{wl_a}{1-\alpha-\beta} + \frac{wl_m}{1-\alpha}} = \frac{(1-\alpha)(1-\alpha-\beta)}{(1-\alpha-\beta)+\beta\psi_a} \tag{34}$$

■

Proof of Lemma 4

Using equation (10) and the fact that all household are identical ($\widetilde{l_p} = l_p$), it follows that:

$$\phi(l_p) = \frac{g'(l_p)(1-l_p)}{[1-g(l_p)]} = \lambda \tag{35}$$

The assumption that $g(1) < 1$ implies that $\phi(1) = 0$. Assumptions $g'(0) \geq 1$ and $g(0) = 0$ imply that $\phi(0) = g'(0) \geq 1$. Furthermore, $\phi'(l_p) = \frac{g"(l_p)(1-l_p)-g'(l_p)[1-\phi(l_p)]}{[1-g(l_p)]} < 0$. Thus, it follows from the Implicit Function Theorem that there is a continuos differentiable function such that:

$$\frac{\partial l_p}{\partial \lambda} = \frac{1}{\frac{g"(l_p)(1-l_p)-g'(l_p)[1-\phi(l_p)]}{[1-g(l_p)]}} < 0 \tag{36}$$

■

Proof of Proposition 5

Combining factors and agriculture goods markets clearing conditions, and equations (14), (15), (17),(18) and (34) we obtain:

$$\psi_a = \frac{(1-\alpha-\lambda)(1-\alpha-\beta)}{\beta\lambda} \tag{37}$$

$$\overline{c} = y_a = \Gamma_a z^\beta \left(\frac{1-\alpha-\beta}{\lambda}\right)^{1-\alpha-\beta} \left(\frac{1-\alpha-\lambda}{\beta}\right)^{1-\beta} k^\alpha l^{1-\alpha-\beta} \tag{38}$$

$$y_m = \Gamma_m \left(\frac{1-\alpha}{\lambda}\right)^{1-\alpha} \frac{[\lambda-(1-\alpha-\beta)]}{\beta} k^\alpha l^{1-\alpha} \tag{39}$$

$$k_m = \frac{[\lambda-(1-\alpha-\beta)]}{\beta} k \tag{40}$$

Using equations (38) and (7) we obtain:

$$\frac{\overline{c}}{\Gamma_a z^\beta k^\alpha} = \left(\frac{1-\alpha-\beta}{\lambda}\right)^{1-\alpha-\beta} \left(\frac{1-\alpha-\lambda}{\beta}\right)^{1-\beta} (1-l_p(\lambda))^{1-\alpha-\beta} \tag{41}$$

It follows from the Implicit Function Theorem that, (36) and (35):

$$\frac{\partial\lambda}{\partial\left(\frac{\bar{c}}{\Gamma_a z^\beta k^\alpha}\right)} = \frac{-1}{\left(\frac{\bar{c}}{\Gamma_a z^\beta k^\alpha}\right)\left[\frac{1-\alpha-\beta}{\lambda}+\frac{1-\beta}{1-\alpha-\lambda}-\frac{1-\alpha-\beta}{1-l_p}\frac{1}{\frac{-g"(l_p)(1-l_p)+g'(l_p)[1-\phi(l_p)]}{[1-g(l_p)]}}\right]} <$$

$$\frac{-1}{\left(\frac{\bar{c}}{\Gamma_a z^\beta k^\alpha}\right)\left[\frac{1-\alpha-\beta}{\lambda}+\frac{1-\beta}{1-\alpha-\lambda}-\frac{1-\alpha-\beta}{1-l_p}\frac{1}{\frac{g'(l_p)[1-\phi(l_p)]}{[1-g(l_p)]}}\right]} =$$

$$\frac{-1}{\left(\frac{\bar{c}}{\Gamma_a z^\beta k^\alpha}\right)\left[\frac{1-\alpha-\beta}{\lambda}+\frac{1-\beta}{1-\alpha-\lambda}-\frac{1-\alpha-\beta}{\lambda(1-\lambda)}\right]} =$$

$$\frac{-(1-\lambda)(1-\alpha-\lambda)}{\left(\frac{\bar{c}}{\Gamma_a z^\beta k^\alpha}\right)\left[\alpha\left[(1-\beta)+(1-\alpha-\lambda)\right]\right]} < 0$$

Thus, it follows from (37), (36) and (7) that:

$$\frac{\partial\psi_a}{\partial\left(\frac{\bar{c}}{\Gamma_a z^\beta k^\alpha}\right)} = \underbrace{\frac{\partial\psi_a}{\partial\lambda}}_{\ominus}\underbrace{\frac{\partial\lambda}{\partial\left(\frac{\bar{c}}{\Gamma_a z^\beta k^\alpha}\right)}}_{\ominus} > 0; \quad \frac{\partial l_p}{\partial\left(\frac{\bar{c}}{\Gamma_a z^\beta k^\alpha}\right)} = \underbrace{\frac{\partial l_p}{\partial\lambda}}_{\ominus}\underbrace{\frac{\partial\lambda}{\partial\left(\frac{\bar{c}}{\Gamma_a z^\beta k^\alpha}\right)}}_{\ominus} > 0;$$

$$\frac{\partial l}{\partial\left(\frac{\bar{c}}{\Gamma_a z^\beta k^\alpha}\right)} = -\underbrace{\frac{\partial l_p}{\partial\left(\frac{\bar{c}}{\Gamma_a z^\beta k^\alpha}\right)}}_{\ominus} > 0$$

■

13.1 Dynamic System

It follows from equations (40) and (39) that:

$$\delta + r = \alpha\frac{y_m}{k_m} = \alpha\Gamma_m\left(\frac{1-\alpha}{\lambda}\right)^{1-\alpha}\left(\frac{l}{k}\right)^{1-\alpha} \tag{42}$$

Thus, it follows from the above equations, the capital accumulation equation (2) and the Euler equation (11) that:

$$\begin{aligned}\dot{k}(t) &= y_m(k(t)) - c_m(t) - \delta k(t)\\ \frac{\dot{c_m}(t)}{c_m(t)} &= (r(k(t))+\delta)(1-g(l_p(k(t)))) - \delta - \rho\end{aligned}$$

where (see equations 39 and 42)

$$y_m(k) = \Gamma_m \left(\frac{1-\alpha}{\lambda(k)}\right)^{1-\alpha} \frac{[\lambda(k) - (1-\alpha-\beta)]}{\beta} k^{\alpha} [l(k)]^{1-\alpha} \tag{43}$$

$$r(k) = \alpha\Gamma_m \left(\frac{1-\alpha}{\lambda(k)}\right)^{1-\alpha} \left(\frac{l(k)}{k}\right)^{1-\alpha} - \delta \tag{44}$$

where $\lambda(k)$ is defined in the proof of proposition 5 (see 41).

Note that in order that the capital locus $\dot{k}(t) = 0$ to be well defined, consumption in manufactures should be positive when $\dot{k}(t) = 0$:

$$c_m^{\dot{k}=0}(k) = y_m(k) - \delta k \geq 0$$

where $c_m^{\dot{k}=0}(k)$ is the level of consumption that makes per capita capital remains constant ($\dot{k} = 0$). In order to guaranty that the above condition hold, we will focus in the analysis of per capita capital when it is above a certain threshold $k^{\min}$, defined as follows:

$$k^{\min} = \max\{k \in [\underline{k}, k^{ss}] \text{ s. th. } y_m(k) = \delta k\} \tag{45}$$

where k^{ss} is the capital at the steady state (we will prove that such steady state exists and is unique in proposition 6) and $\underline{k}$ is defined as the level of capital such that if all the resources of the economy are devoted to agriculture, the subsistence level of consumption is reached:

$$\underline{k} \overset{Def}{\Leftrightarrow} \overline{c} = \Gamma_a z^{\beta} \underline{k}^{\alpha} l (1-\alpha-\beta)^{1-\alpha-\beta} \Leftrightarrow \underline{k} \equiv \left(\frac{\overline{c}}{\Gamma_a z^{\beta} l (1-\alpha-\beta)^{1-\alpha-\beta}}\right)^{\frac{1}{\alpha}}$$

Proof Proposition 6

It follows from Euler equation (20) and (44) that at the steady state:

$$\delta + \rho = \alpha\Gamma_m \left(\frac{1-\alpha}{\lambda}\right)^{1-\alpha} \left(\frac{1-l_p}{k}\right)^{1-\alpha} (1 - g(l_p)) \tag{46}$$

In order that there exists a positive level of consumption of manufactures at the steady state, the following equation should hold (see equations 19 and 43):

$$\begin{aligned} c_m &= y_m(k^{ss}) - \delta k^{ss} > 0 \Leftrightarrow \\ \Gamma_m &\left(\frac{1-\alpha}{\lambda^{ss}}\right)^{1-\alpha} \frac{[\lambda^{ss} - (1-\alpha-\beta)]}{\beta} \left(\frac{1-l_p^{ss}}{k^{ss}}\right)^{1-\alpha} > \delta \end{aligned} \tag{47}$$

Combining equations (46) and (47), it yields:

$$\frac{[\lambda^{ss}-(1-\alpha-\beta)]}{\beta}\frac{\delta+\rho}{\alpha\left(1-g\left(l_p(\lambda^{ss}\right)\right)} > \delta \tag{48}$$

Note that:

$$\begin{aligned}
&\frac{\partial \ln\left(\frac{[\lambda^{ss}-(1-\alpha-\beta)]}{\beta}\frac{\delta+\rho}{\alpha(1-g(l_p(\lambda^{ss}))}\right)}{\partial\lambda} = \\
&\frac{1}{[\lambda^{ss}-(1-\alpha-\beta)]}-\frac{g'\left(l_p(\lambda^{ss}\right)}{\left(1-g\left(l_p(\lambda^{ss}\right)\right)}\frac{1}{\frac{-g"(l_p)(1-l_p)+g'(l_p)[1-\phi(l_p)]}{[1-g(l_p)]}} > \\
&\frac{1}{[\lambda^{ss}-(1-\alpha-\beta)]}-\frac{g'\left(l_p(\lambda^{ss}\right)}{\left(1-g\left(l_p(\lambda^{ss}\right)\right)}\frac{1}{\frac{g'(l_p)[1-\phi(l_p)]}{[1-g(l_p)]}} = \\
&\frac{1}{[\lambda^{ss}-(1-\alpha-\beta)]}-\frac{1}{1-\lambda^{ss}} = \\
&\frac{1+(1-\alpha-\beta)}{[\lambda^{ss}-(1-\alpha-\beta)]\left(1-\lambda^{ss}\right)} > 0
\end{aligned}$$

Note that:

$$\text{if } \lambda = 1-\alpha-\beta \text{ then } \frac{[\lambda^{ss}-(1-\alpha-\beta)]}{\beta}\frac{\delta+\rho}{\alpha\left(1-g\left(l_p(\lambda^{ss}\right)\right)}-\delta = -\delta < 0$$

$$\text{if } \lambda = 1-\alpha \text{ then}$$

$$\frac{[\lambda^{ss}-(1-\alpha-\beta)]}{\beta}\frac{\delta+\rho}{\alpha\left(1-g\left(l_p(\lambda^{ss}\right)\right)}-\delta = \frac{\delta+\rho}{\alpha\left(1-g\left(l_p(\lambda^{ss}\right)\right)}-\delta > 0$$

Thus, it is possible to define:

$$\lambda^{\min ss}(\delta,\rho) \Leftrightarrow \frac{\left[\lambda^{\min ss}(\delta,\rho)-(1-\alpha-\beta)\right]}{\beta}\frac{\delta+\rho}{\alpha\left(1-g\left(l_p(\lambda^{\min ss}(\delta,\rho)\right)\right)} = \delta$$

Thus, in order that (47) holds, the following equation should be satisfied:

$$\lambda > \lambda^{\min ss}(\delta,\rho)$$

Combining equations (41) and (46), it yields:

$$F\left(\lambda, \frac{\overline{c}}{\Gamma_a z^\beta}\left(\frac{\delta+\rho}{\Gamma_m}\right)^{\frac{\alpha}{1-\alpha}}\right) =$$

$$\left(\frac{1-\alpha-\lambda}{\beta\lambda}\right)^{1-\beta}(1-\alpha-\beta)^{1-\beta-\alpha}(1-l_p(\lambda))^{1-\beta}\left(\alpha\left[1-g(l_p(\lambda))\right]\right)^{\frac{\alpha}{1-\alpha}}(1-\alpha)^\alpha$$

$$-\frac{\overline{c}}{\Gamma_a z^\beta}\left(\frac{\delta+\rho}{\Gamma_m}\right)^{\frac{\alpha}{1-\alpha}} = 0 \qquad (49)$$

The derivative of the above equation with respect to λ is as follows:

$$\frac{\frac{\partial F\left(\lambda, \frac{\overline{c}}{\Gamma_a z^\beta}\left(\frac{\delta+\rho}{\Gamma_m}\right)^{\frac{\alpha}{1-\alpha}}\right)}{\partial\lambda}}{\frac{\overline{c}}{\Gamma_a z^\beta}\left(\frac{\delta+\rho}{\Gamma_m}\right)^{\frac{\alpha}{1-\alpha}}} =$$

$$\left[-(1-\beta)\left[\frac{1}{\lambda}+\frac{1}{1-\alpha-\lambda}\right]+\frac{\left[(1-\beta)+\frac{\alpha\lambda}{1-\alpha}\right]}{1-l_p(\lambda)}\frac{1}{\left[-\frac{g"(l_p(\lambda))}{g'(l_p(\lambda))}+\frac{1-\lambda}{1-l_p(\lambda)}\right]}\right] <$$

$$\left[-(1-\beta)\left[\frac{1}{\lambda}+\frac{1}{1-\alpha-\lambda}\right]+\frac{\left[(1-\beta)+\frac{\alpha\lambda}{1-\alpha}\right]}{1-\lambda}\right] =$$

$$\left[-(1-\beta)\left[\frac{1}{\lambda}+\frac{\alpha}{(1-\lambda)}\left[\frac{1}{(1-\alpha-\lambda)}-\frac{\lambda}{(1-\alpha)(1-\beta)}\right]\right]\right] <$$

$$\left[-(1-\beta)\left[\frac{1}{\lambda}+\frac{\alpha}{(1-\lambda)}\left[\frac{1}{(1-\alpha-(1-\alpha-\beta))}-\frac{(1-\alpha)}{(1-\alpha)(1-\beta)}\right]\right]\right] =$$

$$\left[-(1-\beta)\left[\frac{1}{\lambda}+\frac{\alpha}{(1-\lambda)}\left[\frac{1}{\beta}-\frac{1}{(1-\beta)}\right]\right]\right] < 0$$

where in the second inequality we used the fact that $\lambda \in ((1-\alpha-\beta),(1-\alpha))$, and in the last inequality we used the assumption that $\beta \leq \frac{1}{2}$.

Note that $\lambda \in ((1-\alpha-\beta),(1-\alpha))$. Furthermore:

$$\frac{F\left((1-\alpha), \frac{\overline{c}}{\Gamma_a z^\beta}\left(\frac{\delta+\rho}{\Gamma_m}\right)^{\frac{\alpha}{1-\alpha}}\right)}{(\delta+\rho)^{\frac{\alpha}{1-\alpha}}} = -\frac{\overline{c}}{\Gamma_a z^\beta}\left(\frac{1}{\Gamma_m}\right)^{\frac{\alpha}{1-\alpha}} < 0$$

$$\frac{F\left(\lambda^{\min ss}(\delta,\rho), \frac{\overline{c}}{\Gamma_a z^\beta}\left(\frac{\delta+\rho}{\Gamma_m}\right)^{\frac{\alpha}{1-\alpha}}\right)}{(\delta+\rho)^{\frac{\alpha}{1-\alpha}}} = \Omega(\delta,\rho)-\frac{\overline{c}}{\Gamma_a z^\beta}\left(\frac{1}{\Gamma_m}\right)^{\frac{\alpha}{1-\alpha}}$$

where

$$\Omega(\delta,\rho) \equiv \frac{\left(\frac{1}{\lambda^{\min ss}(\delta,\rho)}\right)^{\alpha} (1\text{-}l_p(\lambda^{\min ss}(\delta,\rho)))^{1-\beta} \left(\alpha \left[1\text{-}g(l_p(\lambda^{\min ss}(\delta,\rho)))\right]\right)^{\frac{\alpha}{1-\alpha}} (1\text{-}\alpha)^{\alpha}}{(\delta+\rho)^{\frac{\alpha}{1-\alpha}}} \tag{50}$$

A necessary and sufficient condition in order that there exists a steady state equilibrium with a positive amount of consumption of manufactures is that $\frac{\bar{c}}{\Gamma_a z^{\beta}}\left(\frac{1}{\Gamma_m}\right)^{\frac{\alpha}{1-\alpha}} < \Omega(\delta,\rho)$.

13.2 Dynamic behavior in the surrounding of the steady state:

Let's define $F^{c_m}(k)$ as follows:

$$F^{c_m}(k) = (r(k)+\delta)(1-g(l_p(\lambda(k(t)))) - \delta - \rho$$

Equation (20) may be rewritten as follows:

$$\frac{\dot{c_m}(t)}{c_m(t)} = F^{c_m}(k)$$

Let´s define $k^{\min ss}(\delta,\rho) \overset{Def}{\Leftrightarrow} \lambda\left(k^{\min ss}(\delta,\rho)\right) = \lambda^{\min ss}(\delta,\rho)$.

$$F^{c_m}(k^{\min ss}) =$$
$$\left(\frac{1-\alpha}{\lambda^{\min ss}(\delta,\rho)}\right)^{1-\alpha} \left(\frac{(1-l_p(\lambda^{\min ss}(\delta,\rho)))^{1-\beta}}{\left(\frac{\bar{c}}{\Gamma_a z^{\beta}}\right)\left(\frac{1}{\Gamma_m}\right)^{\frac{\alpha}{1-\alpha}}}\right)^{\frac{1-\alpha}{\alpha}} \left(1-g\left(l_p\left(\lambda^{\min ss}(\delta,\rho)\right)\right)\right)$$
$$-(\delta+\rho) >$$
$$\left(\frac{1-\alpha}{\lambda^{\min ss}(\delta,\rho)}\right)^{1-\alpha} \left(\frac{l\left(\lambda^{\min ss}(\delta,\rho)\right)^{1-\beta}}{\Omega(\delta,\rho)}\right)^{\frac{1-\alpha}{\alpha}} \left(1-g\left(l_p\left(\lambda^{\min ss}(\delta,\rho)\right)\right)\right)$$
$$-(\delta+\rho) = 0$$

Since there is a unique steady state (a unique point in which $F^{c_m}(k^{ss}) = 0$), it follows from continuity that $\forall k \in \left(k^{\min ss}, k^{ss}\right)$ $F^{c_m}(k) > 0$ and $F^{c_m}(k^{ss}) = 0$. Thus, generically $\frac{\partial F^{c_m}(k^{ss})}{\partial k} < 0$.

Thus, if we linearize the Dynamic System (19)-(20) in the surrounding of the steady state we get:

$$\begin{bmatrix} \dot{k}(t) \\ \dot{c_m}(t) \end{bmatrix} = \begin{bmatrix} \frac{\partial y_m(k)}{\partial k} - \delta & -1 \\ \frac{\partial F^{c_m}(k^{ss})}{\partial k} c_m^{ss} & 0 \end{bmatrix} \begin{bmatrix} k(t) - k^{ss} \\ c_m(t) - c_m^{ss} \end{bmatrix}$$

where

$$\frac{\partial F^{c_m}(k^{ss})}{\partial k} < 0 \tag{51}$$

The characteristic equation associated to the above dynamic system is as follows:

$$\begin{vmatrix} \frac{\partial y_m(k)}{\partial k} - \delta - \lambda & -1 \\ \frac{\partial F^{c_m}(k^{ss})}{\partial k} c_m^{ss} & -\lambda \end{vmatrix} =$$

$$\lambda^2 - \left(\frac{\partial y_m(k)}{\partial k} - \delta \right) \lambda + \frac{\partial F^{c_m}(k^{ss})}{\partial k} c_m^{ss} = 0$$

Thus, the roots associated with the above linear equations system are:

$$\lambda = \frac{\left(\frac{\partial y_m(k)}{\partial k} - \delta \right) \pm \sqrt{\left(\frac{\partial y_m(k)}{\partial k} - \delta \right)^2 - 4 \frac{\partial F^{c_m}(k^{ss})}{\partial k} c_m^{ss}}}{2}$$

Since $\frac{\partial F^{c_m}(k^{ss})}{\partial k} c_m^{ss}$ is negative at the steady state, the roots are real ones, being one of them positive and the another one negative. That is, the steady state is a saddle point.

Proof Proposition 7: It follows from (49) that the labor share at the steady state, λ^{ss}, is a decreasing function of Γ_a. Since l_p is a decreasing function of λ (see 36), it follows that l_p^{ss}. Since ψ_a is a decreasing function of λ (see 37), it follows that ψ_a^{ss} is a decreasing function of λ. ■

The Proofs of sections 9 and 10 are available at the "supplementary material" provided by the authors.

14 Supplementary Material: Proofs of sections 9 and 10

Proof Proposition 8:

It follows from equations (5), (6), (14), (17), and (22) that.

$$\begin{aligned}
\lambda &= \frac{wl}{y} = \frac{wl}{p_a y_a + y_m} = \frac{wl}{\frac{wl_a}{1-\alpha-\beta} + \frac{wl_m}{1-\alpha}} = \frac{(1-\alpha)(1-\alpha-\beta)}{(1-\alpha)\frac{l_a}{l} + (1-\alpha-\beta)\frac{l_m}{l}} \\
\lambda &= \frac{(1-\alpha)(1-\alpha-\beta)}{(1-\alpha)\psi_a + (1-\alpha-\beta)(1-\psi_g-\psi_a)} = \frac{(1-\alpha)(1-\alpha-\beta)}{(1-\alpha-\beta)(1-\psi_g) + \beta\psi_a} \\
\lambda &= \frac{(1\text{-}\alpha)(1\text{-}\alpha\text{-}\beta)}{(1\text{-}\alpha\text{-}\beta)(1\text{-}\frac{\vartheta\tau}{\lambda}) + \beta\psi_a} \Rightarrow \psi_a = \frac{1}{\beta}\left[\frac{(1\text{-}\alpha\text{-}\lambda\text{+}\vartheta\tau)(1\text{-}\alpha\text{-}\beta)}{\lambda}\right] \qquad (52)
\end{aligned}$$

where $\psi_a \equiv \frac{l_a}{l}$, $\psi_g \equiv \frac{l_g}{l}$ are respectively the portion of productive labor used in agriculture and hired by the government. Note that the government budget constraint (22) and the definition of λ implies that $\psi_g = \frac{\vartheta\tau}{\lambda}$ .We used in the third equality equations (5), (6), (14) and(17), and in the seventh equality equation (22). Note also that the labor market clearing condition (23) implies that $\frac{l_m}{l} = 1 - \psi_a - \psi_g$.

Since $\psi_a \in \left(0, 1-\psi_g\right) = \left(0, 1-\frac{\vartheta\tau}{\lambda}\right)$ and the labor share is decreasing in ψ_a, it follows from (52) that:

$$\begin{aligned}
\lambda &> \frac{(1-\alpha)(1-\alpha-\beta)}{(1-\alpha-\beta)(1-\frac{\vartheta\tau}{\lambda}) + \beta \times (1-\frac{\vartheta\tau}{\lambda})} \Leftrightarrow \lambda > (1-\alpha-\beta+\vartheta\tau) \\
\lambda &< \frac{(1-\alpha)(1-\alpha-\beta)}{(1-\alpha-\beta)(1-\frac{\vartheta\tau}{\lambda}) + \beta \times 0} \Leftrightarrow \lambda < (1-\alpha+\vartheta\tau) \\
\lambda &\in ((1-\alpha-\beta+\vartheta\tau), (1-\alpha+\vartheta\tau)) \qquad (53)
\end{aligned}$$

Lemma 14 *If $\lambda \in ((1-\alpha-\beta+\vartheta\tau), (1-\alpha+\vartheta\tau))$, there is $\widetilde{\tau} \in (0,1)$ (defined bellow) such that if $\tau \leq \widetilde{\tau}$ then $\frac{\partial l_p}{\partial \lambda} \in \left(-\frac{1-l_p}{\lambda}, 0\right)$. Furthermore, $l_p < \frac{1}{2}$.*

It follows from (21) that at equilibrium:

$$\frac{g'_{l_p}(l_p)\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)(1-l_p)}{\left[1-\tau-g(l_p)\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)\right]}-\lambda=0 \quad (54)$$

$$\frac{\partial l_p}{\partial\lambda}=\frac{1-l_p}{\lambda}\times \frac{-\frac{\varphi'_{l_g}\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)\frac{\vartheta\tau}{\lambda}(1-l_p)}{\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)}\left[\frac{1-\tau}{1-\tau-g(l_p)\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)}\right]-1}{-\frac{g"_{l_p}(l_p)l_p}{g'_{l_p}(l_p)}\frac{1-l_p}{l_p}+1-\lambda+\frac{\varphi'_{l_g}\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)\frac{\vartheta\tau}{\lambda}(1-l_p)}{\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)}\frac{1-\tau}{1-\tau-g(l_p)\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)}} \quad (55)$$

Note that if $\tau=0$ equation (54) and the assumption that $\frac{g'_{l_p}\left(\frac{1}{2}\right)\frac{1}{2}}{1-g\left(\frac{1}{2}\right)}-1-\alpha-\beta>0$ implies that:

$$\text{if } \tau=0\ \frac{g'_{l_p}(\frac{1}{2})\varphi\left(\frac{\vartheta\times 0}{\lambda}(1-\frac{1}{2}),\xi\right)(1-\frac{1}{2})}{\left[1-0-g(\frac{1}{2})\varphi\left(\frac{\vartheta\times 0}{\lambda}(1-\frac{1}{2}),\xi\right)\right]}=\frac{g'_{l_p}\left(\frac{1}{2}\right)\frac{1}{2}}{1-g\left(\frac{1}{2}\right)}>1-\alpha-\beta>\lambda\Rightarrow l_p<\frac{1}{2}$$

It follows from the assumption that $\frac{g'_{l_p}\left(\frac{1}{2}\right)\frac{1}{2}}{1-g\left(\frac{1}{2}\right)}-1-\alpha-\beta>0$ that it is possible to define $\overline{\tau}_1$ as follows: $\overline{\tau}_1=\min\left\{\tau \text{ such that } \max\limits_{\lambda\in[(1-\alpha-\beta+\vartheta\tau),(1-\alpha+\vartheta\tau)]}\left[\frac{g'_{l_p}\left(\frac{1}{2}\right)\varphi\left(\frac{\vartheta\tau}{\lambda}\frac{1}{2},\min\xi\right)\frac{1}{2}}{\left[1-\tau-g\left(\frac{1}{2}\right)\varphi\left(\frac{\vartheta\tau}{\lambda}\frac{1}{2},\min\xi\right)\right]}-\lambda\right]=0\right\}$. We assume that $\tau\le\overline{\tau}_1$. This assumption implies that $l_p\le\frac{1}{2}$. It follows from the assumption $\min\limits_{l_p\in\left[0.\frac{1}{2}\right]}-\varepsilon_{l_p}^{g'(l_p)}(l_p)>1-\alpha$ that it is possible to define $\overline{\tau}_2$ as follows $\overline{\tau}_2\overset{Def}{\Leftrightarrow}\min\limits_{l_p\in\left[1.\frac{1}{2}\right]}-\varepsilon_{l_p}^{g'(l_p)}(l_p)=1-\alpha+\vartheta\overline{\tau}_2$. We assume that $\tau\le\overline{\tau}_2$. This assumption implies that $\min\limits_{l_p\in\left[1.\frac{1}{2}\right]}-\varepsilon_{l_p}^{g'(l_p)}(l_p)\ge 1-\alpha+\vartheta\overline{\tau}_2>\lambda$ (see 53). The above assumptions implies that $l_p\le\frac{1}{2}$ and $\min\limits_{l_p\in\left[1.\frac{1}{2}\right]}-\varepsilon_{l_p}^{g'(l_p)}(l_p)>\lambda$. Thus:

$$-\frac{g"_{l_p}(l_p)l_p}{g'_{l_p}(l_p)}\frac{1-l_p}{l_p}\ \ge\ -\frac{g"_{l_p}(l_p)l_p}{g'_{l_p}(l_p)}\frac{1-\frac{1}{2}}{\frac{1}{2}}\ \ge\ \min_{l_p\in\left[1.\frac{1}{2}\right]}-\varepsilon_{l_p}^{g'(l_p)}(l_p)\ >\ \lambda$$

$$\Rightarrow\ -\frac{g"_{l_p}(l_p)l_p}{g'_{l_p}(l_p)}\frac{1-l_p}{l_p}-\lambda>0 \quad (56)$$

This implies that:

$$\frac{\partial\left(\frac{x-1}{-\frac{g"_{l_p}(l_p)l_p}{g'_{l_p}(l_p)}\frac{1-l_p}{l_p}+1-\lambda-x}\right)}{\partial x}=\frac{-\frac{g"_{l_p}(l_p)l_p}{g'_{l_p}(l_p)}\frac{1-l_p}{l_p}-\lambda}{\left[-\frac{g"_{l_p}(l_p)l_p}{g'_{l_p}(l_p)}\frac{1-l_p}{l_p}+1-\lambda-x\right]^2}>0$$

From the previous derivative it follows that the function $\frac{x-1}{-\frac{g"_{l_p}(l_p)l_p}{g'_{l_p}(l_p)}\frac{1-l_p}{l_p}+1-\lambda-x}$ is increasing in x. Thus:

$$\frac{-\frac{\varphi'_{l_g}\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)\frac{\vartheta\tau}{\lambda}(1-l_p)}{\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)}\left[\frac{1-\tau}{\left[1-\tau-g(l_p)\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)\right]}\right]-1}{-\frac{g"_{l_p}(l_p)l_p}{g'_{l_p}(l_p)}\frac{1-l_p}{l_p}+1-\lambda-\left[-\frac{\varphi'_{l_g}\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)\frac{\vartheta\tau}{\lambda}(1-l_p)}{\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)}\left[\frac{1-\tau}{\left[1-\tau-g(l_p)\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)\right]}\right]\right]}> \frac{-1}{-\frac{g"_{l_p}(l_p)l_p}{g'_{l_p}(l_p)}\frac{1-l_p}{l_p}+1-\lambda} \quad (57)$$

Thus, it follows from (56) and (57) that:

$$\frac{\partial l_p}{\partial\lambda}=\frac{1-l_p}{\lambda}\frac{-\frac{\varphi'_{l_g}\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)\frac{\vartheta\tau}{\lambda}(1-l_p)}{\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)}\left[\frac{1-\tau}{\left[1-\tau-g(l_p)\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)\right]}\right]-1}{-\frac{g"_{l_p}(l_p)l_p}{g'_{l_p}(l_p)}\frac{1-l_p}{l_p}+1-\lambda+\frac{\varphi'_{l_g}\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)\frac{\vartheta\tau}{\lambda}(1-l_p)}{\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)}\frac{1-\tau}{1-\tau-g(l_p)\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)}}> \frac{1-l_p}{\lambda}\frac{-1}{-\frac{g"_{l_p}(l_p)l_p}{g'_{l_p}(l_p)}\frac{1-l_p}{l_p}+1-\lambda}>-\frac{1-l_p}{\lambda} \quad (58)$$

Note that $\lim_{\tau\to 0}-\frac{\varphi'_{l_g}\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)\frac{\vartheta\tau}{\lambda}(1-l_p)}{\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)}\left[\frac{1-\tau}{1-\tau-g(l_p)\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)}\right]=0$. Thus, it is possible to define $\overline{\tau}_3$ as follows:

$$\overline{\tau}_3 \overset{Def}{\equiv} \min\left\{\tau \text{ such that } \max_{l_p\in\left[0,\frac{1}{2}\right],\lambda\in[(1-\alpha-\beta+\vartheta\tau),(1-\alpha+\vartheta\tau)]}-\frac{\varphi'_{l_g}\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)\frac{\vartheta\tau}{\lambda}(1-l_p)}{\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)}\left[\frac{1-\tau}{\left[1-\tau-g(l_p)\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)\right]}\right]=1\right\}.$$

Thus, if $\tau\leq\overline{\tau}_3$ then $\frac{\partial l_p}{\partial\lambda}\leq 0$. Let's define $\widetilde{\tau}=\min\{\overline{\tau}_1,\overline{\tau}_2,\overline{\tau}_3\}$. Q.E.D. Lemma■

We will assume from now on that $\tau \leq \widetilde{\tau}$.

Using agriculture goods clearing condition and equations (14), (15), (17),(18), (52), (22) and (23) we obtain:

$$\left(\frac{1-\alpha+\vartheta\tau-\lambda}{\beta}\right)^{1-\beta}\left[\frac{1-\alpha-\beta}{\lambda}\left(1-\frac{\vartheta\tau}{\lambda}\right)(1-l_p)\right]^{1-\alpha-\beta}-\frac{\overline{c}}{\Gamma_a z^\beta k^\alpha}=0 \tag{59}$$

It follows from Euler equation and equations (17),(18), (52), (22) and (23) that the following equation should hold at the steady state:

$$\left(\frac{\alpha\Gamma_m\left[1-\tau-g(l_p)\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)\right]}{(\delta+\rho)}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\lambda}\right)\left(1-\frac{\vartheta\tau}{\lambda}\right)(1-l_p)=k \tag{60}$$

Combining equations (59) and (60):

$$\begin{aligned}
&F\left(\lambda,\frac{\overline{c}}{\Gamma_a z^\beta}\left(\frac{\delta+\rho}{\Gamma_m}\right)^{\frac{\alpha}{1-\alpha}},\tau,\vartheta,\xi\right)=\\
&\ln\left[\left(\frac{1-\alpha+\vartheta\tau-\lambda}{\beta\lambda}\right)^{1-\beta}(1-\alpha-\beta)^{1-\alpha-\beta}\left(1-\frac{\vartheta\tau}{\lambda}\right)^{1-\beta}(1-l_p(\lambda,\vartheta,\tau,\xi)))^{1-\beta}\times\right.\\
&\left.\left[\alpha\left[1-\tau-g(l_p(\lambda,\vartheta,\tau,\xi))\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p(\lambda,\vartheta,\tau,\xi)),\xi\right)\right]\right]^{\frac{\alpha}{1-\alpha}}(1-\alpha)^\alpha\right]\\
&-\ln\left[\frac{\overline{c}}{\Gamma_a z^\beta}\left(\frac{\delta+\rho}{\Gamma_m}\right)^{\frac{\alpha}{1-\alpha}}\right]=0
\end{aligned}$$

The derivative of the above equation with respect to λ is as follows:

$$\frac{\partial F\left(\lambda,\frac{\overline{c}}{\Gamma_a z^\beta}\left(\frac{\delta+\rho}{\Gamma_m}\right)^{\frac{\alpha}{1-\alpha}},\tau,\vartheta,\xi\right)}{\partial\lambda}=$$

$$-(1-\beta)\left[\frac{1}{\lambda}+\frac{1}{1-\alpha+\vartheta\tau-\lambda}\right]-\frac{\left[(1-\beta)+\frac{\alpha\lambda}{1-\alpha}\right]}{1-l_p}\frac{\partial l_p}{\partial\lambda}$$

$$-\frac{g(l_p(\lambda,\vartheta,\tau,\xi))\varphi'_{l_g}\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)\frac{\vartheta\tau}{\lambda}(1-l_p)}{1-\tau-g(l_p(\lambda,\vartheta,\tau,\xi))\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)}\left[\frac{1}{\lambda}-\frac{\partial l_p}{\partial\lambda}\right]+\frac{(1-\beta)\vartheta\tau}{\lambda(\lambda-\vartheta\tau)}<$$

$$-(1-\beta)\left[\frac{1}{\lambda}+\frac{1}{1-\alpha+\vartheta\tau-\lambda}\right]+\frac{\left[(1-\beta)+\frac{\alpha\lambda}{1-\alpha}\right]}{1-l_p}\frac{1-l_p}{\lambda}+\frac{(1-\beta)\vartheta\tau}{\lambda(\lambda-\vartheta\tau)}=$$

$$-\frac{1-\beta}{1-\alpha+\vartheta\tau-\lambda}+\frac{\alpha}{1-\alpha}+\frac{(1-\beta)\vartheta\tau}{\lambda(\lambda-\vartheta\tau)}<$$

$$-\frac{1\text{-}\beta}{1\text{-}\alpha+\vartheta\tau\text{-}\frac{(1-\alpha+\vartheta\tau)(1-\alpha-\beta)}{(1-\alpha)}}+\frac{\alpha}{1\text{-}\alpha}+\frac{(1\text{-}\beta)\vartheta\tau}{\frac{(1-\alpha+\vartheta\tau)(1-\alpha-\beta)}{(1-\alpha)}\left(\frac{(1-\alpha+\vartheta\tau)(1-\alpha-\beta)}{(1-\alpha)}\text{-}\vartheta\tau\right)}=$$

$$\frac{-\frac{(1-\beta-\alpha)(1-\alpha)-\beta\alpha\vartheta\tau}{\beta(1-\alpha)}+\frac{(1-\alpha)^2(1-\beta)\vartheta\tau}{(1-\alpha-\beta)[(1-\alpha)(1-\alpha-\beta)-\beta\vartheta\tau]}}{(1-\alpha+\vartheta\tau)}$$

where we have used (58)Let's define $\overline{\tau}_4$ as follows: $\overline{\tau}_4\overset{Def}{\Leftrightarrow}-\frac{(1-\beta-\alpha)(1-\alpha)-\beta\alpha\vartheta\overline{\tau}_4}{\beta(1-\alpha)}+\frac{(1-\alpha)^2(1-\beta)\vartheta\overline{\tau}_4}{(1-\alpha-\beta)[(1-\alpha)(1-\alpha-\beta)-\beta\vartheta\overline{\tau}_4]}=0$. We assume that $\tau\le\overline{\tau}_4$. Thus:

$$\frac{\partial F\left(\lambda,\frac{\overline{c}}{\Gamma_a z^\beta}\left(\frac{\delta+\rho}{\Gamma_m}\right)^{\frac{\alpha}{1-\alpha}},\tau,\vartheta,\xi\right)}{\partial\lambda}<0$$

Definition 15 $\overline{\tau}=\min\{\overline{\tau}_1,\overline{\tau}_2,\overline{\tau}_3,\overline{\tau}_4\}$ *where*

$$\overline{\tau}_1=\min\left\{\tau \textit{ such that } \max_{\lambda\in\left[\frac{(1-\alpha+\vartheta\tau)(1-\alpha-\beta)}{(1-\alpha)},(1-\alpha+\vartheta\tau)\right]}\left[\frac{g'_{l_p}\left(\frac{1}{2}\right)\varphi\left(\frac{\vartheta\tau}{\lambda}\frac{1}{2},\min\xi\right)\frac{1}{2}}{\left[1-\tau-g\left(\frac{1}{2}\right)\varphi\left(\frac{\vartheta\tau}{\lambda}\frac{1}{2},\min\xi\right)\right]}-\lambda\right]=0\right\};$$

$$\overline{\tau}_2\overset{Def}{\Leftrightarrow}\min_{l_p\in\left[1.\frac{1}{2}\right]}-\varepsilon_{l_p}^{g'(l_p)}(l_p)=1-\alpha+\vartheta\overline{\tau}_2;$$

$$\overline{\tau}_3\overset{Def}{\equiv}\min\left\{\tau \textit{ such that } \max_{l_p\in\left[0,\frac{1}{2}\right],\lambda\in[(1-\alpha-\beta+\vartheta\tau),(1-\alpha+\vartheta\tau)]}-\frac{\varphi'_{l_g}\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)\frac{\vartheta\tau}{\lambda}(1-l_p)}{\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)}\left[\frac{1-\tau}{\left[1-\tau-g(l_p)\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)\right]}\right]=1\right\}$$

$\overline{\tau}_4 \overset{Def}{\Leftrightarrow} -\frac{(1-\beta-\alpha)(1-\alpha)-\beta\alpha\vartheta\overline{\tau}_4}{\beta(1-\alpha)} + \frac{(1-\alpha)^2(1-\beta)\vartheta\overline{\tau}_4}{(1-\alpha-\beta)[(1-\alpha)(1-\alpha-\beta)-\beta\vartheta\overline{\tau}_4]} = 0.$

We assume that $\tau \leq \overline{\tau}$. This assumption implies that $l_p \leq \frac{1}{2}$, that $\frac{\partial l_p}{\partial \lambda} \in \left(-\frac{1-l_p}{\lambda}, 0\right]$ and $\frac{\partial F(.)}{\partial \lambda} < 0$.

Note that:

$$F\left(\lambda, \frac{\overline{c}}{\Gamma_a z^\beta}\left(\frac{\delta+\rho}{\Gamma_m}\right)^{\frac{\alpha}{1-\alpha}}, \tau, \vartheta, \xi\right) = \ln\left[h(\lambda)\right] - \ln\left[\frac{\overline{c}}{\Gamma_a z^\beta}\left(\frac{\delta+\rho}{\Gamma_m}\right)^{\frac{\alpha}{1-\alpha}}\right] = 0$$

where

$h(\lambda,\tau,\vartheta,\xi) \equiv \left(\frac{1-\alpha+\vartheta\tau-\lambda}{\beta\lambda}\right)^{1-\beta}(1-\alpha-\beta)^{1-\alpha-\beta}\left(1-\frac{\vartheta\tau}{\lambda}\right)^{1-\beta}(1-l_p(\lambda,\vartheta,\tau,\xi)))^{1-\beta}\times$ $\left[\alpha\left[1-\tau-g(l_p(\lambda,\vartheta,\tau,\xi))\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p(\lambda,\vartheta,\tau,\xi)),\xi\right)\right]\right]^{\frac{\alpha}{1-\alpha}}(1-\alpha)^\alpha$. Note that $h(1-\alpha+\vartheta\tau,\tau,\vartheta,\xi) = 0 < \frac{\overline{c}}{\Gamma_a z^\beta}\left(\frac{\delta+\rho}{\Gamma_m}\right)^{\frac{\alpha}{1-\alpha}}$, A sufficient condition in order that there exist steady state equilibrium is that $\frac{\overline{c}}{\Gamma_a z^\beta}\left(\frac{\delta+\rho}{\Gamma_m}\right)^{\frac{\alpha}{1-\alpha}} < \Omega \equiv \inf_{\tau\in[0,\overline{\tau})} h\left(\frac{(1-\alpha+\vartheta\tau)(1-\alpha-\beta)}{(1-\alpha)}, \tau, \vartheta, \xi\right)$. Note that if $\tau \leq \overline{\tau}$ then:

$$\begin{aligned}
&h\left(\frac{(1-\alpha+\vartheta\tau)(1-\alpha-\beta)}{(1-\alpha)}\right) = \\
&\left(\frac{1}{(1-\alpha-\beta)}\right)^\alpha\left(\frac{(1-\alpha)(1-\alpha-\beta)-\beta\vartheta\tau}{(1-\alpha+\vartheta\tau)(1-\alpha-\beta)}\right)^{1-\beta}\times \\
&\left(1-l_p\left(\frac{(1-\alpha+\vartheta\tau)(1-\alpha-\beta)}{(1-\alpha)},\vartheta,\tau,\xi\right)\right)^{1-\beta}\alpha^{\frac{\alpha}{1-\alpha}}\times \\
&\left[1-\tau-g\left(l_p\left(\frac{(1-\alpha+\vartheta\tau)(1-\alpha-\beta)}{(1-\alpha)},\vartheta,\tau,\xi\right)\right)\times\right. \\
&\left.\varphi\left(\frac{(1-\alpha)\vartheta\tau\left(1-l_p\left(\frac{(1-\alpha+\vartheta\tau)(1-\alpha-\beta)}{(1-\alpha)},\vartheta,\tau,\xi\right)\right)}{(1-\alpha+\vartheta\tau)(1-\alpha-\beta)},\xi\right)\right]^{\frac{\alpha}{1-\alpha}} \\
&(1-\alpha)^\alpha > \\
&\left(\frac{1}{(1-\alpha-\beta)}\right)^\alpha\left(\frac{(1-\alpha)(1-\alpha-\beta)-\beta\vartheta\tau}{(1-\alpha+\vartheta\tau)(1-\alpha-\beta)}\right)^{1-\beta}\times \\
&\left(1-\frac{1}{2}\right)^{1-\beta}\left[\alpha\left[1-\tau-g\left(\frac{1}{2}\right)\right]\right]^{\frac{\alpha}{1-\alpha}}(1-\alpha)^\alpha > 0
\end{aligned}$$

where the last inequality come from definition of τ:

$$\begin{aligned}\Omega \equiv & \inf_{\tau\in[0,\overline{\tau})} h\left(\frac{(1-\alpha+\vartheta\tau)(1-\alpha-\beta)}{(1-\alpha)},\tau,\vartheta,\xi\right) \geq \\ & \left(\frac{1}{(1-\alpha-\beta)}\right)^{\alpha}\left(\frac{(1-\alpha)(1-\alpha-\beta)-\beta\vartheta\tau}{(1-\alpha+\vartheta\tau)(1-\alpha-\beta)}\right)^{1-\beta} \times \\ & \left(1-\frac{1}{2}\right)^{1-\beta}\left[\alpha\left[1-\tau-g\left(\frac{1}{2}\right)\right]\right]^{\frac{\alpha}{1-\alpha}}(1-\alpha)^{\alpha} > 0\end{aligned}$$

Since $F(.)$ is strictly decreasing in λ, it follows that when $\frac{\overline{c}}{\Gamma_a z^{\beta}}\left(\frac{\delta+\rho}{\Gamma_m}\right)^{\frac{\alpha}{1-\alpha}} < \Omega$, there exist a unique steady state.

Applying the Implicit Function Theorem:

$$\frac{\partial\lambda^{ss}}{\partial\left(\frac{\overline{c}}{\Gamma_a z^{\beta}}\left(\frac{\delta+\rho}{\Gamma_m}\right)^{\frac{\alpha}{1-\alpha}}\right)} = -\frac{\frac{\partial F\left(\lambda,\frac{\overline{c}}{\Gamma_a z^{\beta}}\left(\frac{\delta+\rho}{\Gamma_m}\right)^{\frac{\alpha}{1-\alpha}},\tau,\vartheta,\xi\right)}{\partial\left(\frac{\overline{c}}{\Gamma_a z^{\beta}}\left(\frac{\delta+\rho}{\Gamma_m}\right)^{\frac{\alpha}{1-\alpha}}\right)}}{\frac{\partial F\left(\lambda,\frac{\overline{c}}{\Gamma_a z^{\beta}}\left(\frac{\delta+\rho}{\Gamma_m}\right)^{\frac{\alpha}{1-\alpha}},\tau,\vartheta,\xi\right)}{\partial\lambda}} = \frac{-1}{-\frac{\partial F\left(\lambda,\frac{\overline{c}}{\Gamma_a z^{\beta}}\left(\frac{\delta+\rho}{\Gamma_m}\right)^{\frac{\alpha}{1-\alpha}},\tau,\vartheta,\xi\right)}{\partial\lambda}} < 0$$

Therefore:

$$\frac{\partial\lambda^{ss}}{\partial\overline{c}} < 0;\ \frac{\partial\lambda^{ss}}{\partial(\delta+\rho)} < 0;\ \frac{\partial\lambda^{ss}}{\partial\Gamma_a} > 0;\ \frac{\partial\lambda^{ss}}{\partial z} > 0;\ \frac{\partial\lambda^{ss}}{\partial\Gamma_m} > 0 \tag{61}$$

Note that:

$$\begin{aligned}\frac{\partial F\left(\lambda,\frac{\overline{c}}{\Gamma_a z^{\beta}}\left(\frac{\delta+\rho}{\Gamma_m}\right)^{\frac{\alpha}{1-\alpha}},\tau,\vartheta,\xi\right)}{\partial\xi} = & -\frac{\left[(1-\beta)+\frac{\alpha\lambda}{1-\alpha}\right]}{1-l_p}\frac{\partial l_p}{\partial\xi} \\ & -\frac{g(l_p(\lambda,\vartheta,\tau,\xi))\varphi'_{\xi}\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)-g(l_p(\lambda,\vartheta,\tau,\xi))\varphi'_{l_g}\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)\frac{\vartheta\tau}{\lambda}\frac{\partial l_p}{\partial\xi}}{1-\tau-g(l_p(\lambda,\vartheta,\tau,\xi))\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)} > 0\end{aligned}$$

Applying the Implicit Function Theorem, it follows that:

$$\frac{\partial\lambda^{ss}}{\partial\xi} > 0 \tag{62}$$

Note that:

$$\frac{\partial F\left(\lambda, \frac{\overline{c}}{\Gamma_a z^\beta}\left(\frac{\delta+\rho}{\Gamma_m}\right)^{\frac{\alpha}{1-\alpha}}, \tau, \vartheta, \xi\right)}{\partial \vartheta} =$$

$$-\frac{(1-\beta)\frac{\tau}{\lambda}}{\left(1-\frac{\vartheta\tau}{\lambda}\right)} - \frac{\left[(1-\beta)+\frac{\alpha\lambda}{1-\alpha}\right]}{1-l_p}\frac{\partial l_p}{\partial \vartheta}$$

$$-\frac{g(l_p(\lambda,\vartheta,\tau,\xi))\varphi'_{l_g}\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)\frac{\tau}{\lambda}\left[(1-l_p)-\vartheta\frac{\partial l_p}{\partial \vartheta}\right]}{1-\tau-g(l_p(\lambda,\vartheta,\tau,\xi))\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)}$$

where

$$\frac{\partial l_p}{\partial \vartheta} =$$

$$(1-l_p)\frac{\frac{\varphi'_{l_g}\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)\frac{\tau}{\lambda}(1-l_p)}{\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)}\frac{1-\tau}{1-\tau-g(l_p)\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)}}{-\frac{g"_{l_p}(l_p)l_p}{g'_{l_p}(l_p)}\frac{1-l_p}{l_p}+1-\lambda+\frac{\varphi'_{l_g}\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)\frac{\vartheta\tau}{\lambda}(1-l_p)}{\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)}\frac{1-\tau}{1-\tau-g(l_p)\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)}} < 0$$

$$\frac{\partial(1-l_p-l_g)}{\partial \tau} = \frac{\partial\left[(1-l_p)\left(1-\frac{\vartheta\tau}{\lambda}\right)\right]}{\partial \tau} = -\left(1-\frac{\vartheta\tau}{\lambda}\right)\frac{\partial l_p}{\partial \tau} - (1-l_p)\frac{\vartheta}{\lambda}$$

Since $\lim_{l_g\to 0}\varphi'_{l_g}(l_g,\xi) = -\infty$ then $\lim_{\vartheta\to 0}\frac{\frac{\partial l_p}{\partial \vartheta}}{\tau} = -\infty$ and $\lim_{\vartheta\to 0}\frac{\frac{\partial(1-l_p-l_g)}{\partial \vartheta}}{\tau} = \lim_{\vartheta\to 0}\frac{\frac{\partial(1-l_p-l_g)}{\partial \vartheta}}{\tau} = +\infty$, therefore $\lim_{l_g\to 0}\frac{\frac{\partial F\left(\lambda, \frac{\overline{c}}{\Gamma_a z^\beta}\left(\frac{\delta+\rho}{\Gamma_m}\right)^{\frac{\alpha}{1-\alpha}}, \tau, \vartheta, \xi\right)}{\partial \vartheta}}{\tau} = +\infty$. Thus, there is an interval $\tau \in (0, \widetilde{\tau}_\vartheta)$ in which $\frac{\partial l_p}{\partial \vartheta} > 0$; $\frac{\partial(1-l_p-l_g)}{\partial \vartheta} > 0$ and $\frac{\partial F\left(\lambda, \frac{\overline{c}}{\Gamma_a z^\beta}\left(\frac{\delta+\rho}{\Gamma_m}\right)^{\frac{\alpha}{1-\alpha}}, \tau, \vartheta, \xi\right)}{\partial \vartheta} > 0$. Thus, if $\tau \in (0, \widetilde{\tau}_\vartheta)$ $\frac{\partial \lambda^{ss}}{\partial \vartheta} = -\frac{\frac{\partial F\left(\lambda, \frac{\overline{c}}{\Gamma_a z^\beta}\left(\frac{\delta+\rho}{\Gamma_m}\right)^{\frac{\alpha}{1-\alpha}}, \tau, \vartheta, \xi\right)}{\partial \vartheta}}{\frac{\partial F\left(\lambda, \frac{\overline{c}}{\Gamma_a z^\beta}\left(\frac{\delta+\rho}{\Gamma_m}\right)^{\frac{\alpha}{1-\alpha}}, \tau, \vartheta, \xi\right)}{\partial \lambda}} > 0$.

It follows from (54) that:

$$\frac{\partial l_p}{\partial \tau} = \frac{(1-l_p)}{\left[1-\tau-g(l_p)\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)\right]} \times$$
$$\frac{\frac{\varphi'_{l_g}\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)\frac{\vartheta}{\lambda}(1-l_p)}{\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)}[1-\tau]+1}{-\frac{g"_{l_p}(l_p)l_p}{g'_{l_p}(l_p)}\frac{1-l_p}{l_p}+1-\lambda+\frac{\varphi'_{l_g}\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)\frac{\vartheta\tau}{\lambda}(1-l_p)}{\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)}\frac{1-\tau}{1-\tau-g(l_p)\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)}}$$
$$\frac{\partial(1-l_p-l_g)}{\partial\tau} = \frac{\partial\left[(1-l_p)\left(1-\frac{\vartheta\tau}{\lambda}\right)\right]}{\partial\tau} = -\left(1-\frac{\vartheta\tau}{\lambda}\right)\frac{\partial l_p}{\partial\tau}-(1-l_p)\frac{\vartheta}{\lambda}$$
$$\frac{\partial\left[1-\tau-g(l_p)\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)\right]}{\partial\tau} = -1-g'(l_p)\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)\frac{\partial l_p}{\partial\tau}$$
$$-g(l_p)\varphi'_{l_g}\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)\left[\frac{\vartheta}{\lambda}(1-l_p)-\frac{\vartheta\tau}{\lambda}\frac{\partial l_p}{\partial\tau}\right]$$

It follows from the assumption that $\lim_{l_g\to 0}\varphi'_{l_g}(l_g,\xi)=-\infty$, that $\lim_{\tau\to 0}\frac{\partial l_p}{\partial\tau}=-\infty$, $\lim_{\tau\to 0}\frac{\partial(1-l_g-l_p)}{\partial\tau} = \lim_{\tau\to 0}\frac{\partial\left[(1-l_p)\left(1-\frac{\vartheta\tau}{\lambda}\right)\right]}{\partial\tau} = \infty$, $\lim_{l_g\to 0}\frac{\partial\left[1-\tau-g(l_p)\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)\right]}{\partial\tau} = +\infty$. Then it is possible to define $\widehat{\tau}\leq\overline{\tau}$ such that $\forall\tau<\widehat{\tau}$ $\frac{\partial l_p}{\partial\tau}<0$, $\frac{\partial(1-l_g-l_p)}{\partial\tau}>\infty$, and $\frac{\partial\left[1-\tau-g(l_p)\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)\right]}{\partial\tau}>0$. Thus, if $\tau<\widehat{\tau}$ then $\frac{\partial F\left(\lambda,\frac{\overline{c}}{\Gamma_a z^\beta}\left(\frac{\delta+\rho}{\Gamma_m}\right)^{\frac{\alpha}{1-\alpha}},\tau,\vartheta,\xi\right)}{\partial\tau}>0$ and $\frac{\partial\lambda}{\partial\tau}>0$.

It follows from (59) and (54) that:

$$\ln k^\alpha = \frac{\overline{c}}{\Gamma_a z^\beta}\left(\frac{\beta}{1-\alpha+\vartheta\tau-\lambda}\right)^{1-\beta}\left(\frac{\lambda}{\lambda-\vartheta\tau}\right)^{1-\beta-\alpha}\left(\frac{\lambda}{1-l_p}\right)^{1-\beta-\alpha} \quad (63)$$

It follows from (58) that:

$$\frac{\partial\ln\left(\frac{\lambda}{1-l_p}\right)}{\partial\lambda} = \frac{1}{\lambda}+\frac{1}{1-l_p}\frac{\partial l_p}{\partial\lambda} > \frac{1}{\lambda}-\frac{1}{\lambda} = 0$$

Thus:

$$\frac{\partial k}{\partial\lambda}>0 \quad (64)$$

Furthermore, it follows from (60) that:

$$G(k, l_p, \lambda, \vartheta, , \tau, ...) =$$
$$\left(\frac{\alpha \Gamma_m \left[1\text{-}\tau\text{-}g(l_p)\varphi\left(\frac{\vartheta\tau}{\lambda}(1\text{-}l_p), \xi\right)\right]}{(\delta + \rho)} \right)^{\frac{1}{1-\alpha}} \left(\frac{1\text{-}\alpha}{\lambda}\right)\left(1\text{-}\frac{\vartheta\tau}{\lambda}\right)(1\text{-}l_p) - k^{ss}$$

When $\tau \in (0, \widetilde{\tau}_\vartheta)$:

$$\frac{\partial k^{ss}}{\partial \vartheta} = -\frac{\frac{\partial G}{\partial \lambda}\frac{\partial \lambda}{\partial \vartheta} + \frac{\partial G}{\partial \vartheta}}{\frac{\partial G}{\partial k}} = \frac{\frac{\partial G}{\partial \lambda}}{-\frac{\partial G}{\partial k}}\frac{\partial \lambda}{\partial \vartheta} + \frac{\frac{\partial G}{\partial \vartheta}}{-\frac{\partial G}{\partial k}} = \underbrace{\frac{\partial k}{\partial \lambda}}_{+}\underbrace{\frac{\partial \lambda}{\partial \vartheta}}_{+} + \frac{\frac{\partial G}{\partial \vartheta}}{-\frac{\partial G}{\partial k}} =$$
$$\underbrace{\frac{\partial k}{\partial \lambda}}_{+}\underbrace{\frac{\partial \lambda}{\partial \vartheta}}_{+} + \frac{\frac{\partial G}{\partial \vartheta}}{-(-1)} = \underbrace{\frac{\partial k}{\partial \lambda}}_{+}\underbrace{\frac{\partial \lambda}{\partial \vartheta}}_{+} + \frac{\partial G}{\partial \vartheta}$$

Note that:

$$\frac{\frac{\partial G}{\partial \vartheta}}{G(.)} = -\frac{1}{1-\alpha}\frac{g(l_p)\varphi'_{l_g}\left(\frac{\vartheta\tau}{\lambda}(1-l_p), \xi\right)}{1-\tau-g(l_p)\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p), \xi\right)} + \frac{\frac{\partial\left[\left(1-\frac{\vartheta\tau}{\lambda}\right)(1-l_p)\right]}{\partial \vartheta}}{\left(1-\frac{\vartheta\tau}{\lambda}\right)(1-l_p)} =$$
$$-\frac{1}{1-\alpha}\frac{g(l_p)\varphi'_{l_g}\left(\frac{\vartheta\tau}{\lambda}(1-l_p), \xi\right)}{1-\tau-g(l_p)\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p), \xi\right)} + \frac{\frac{\partial[(1-l_p-l_g)]}{\partial \vartheta}}{1-l_p-l_g}$$

We have shown that for $\tau \in (0, \widetilde{\tau}_\vartheta)$ $\frac{\partial[(1-l_p-l_g)]}{\partial \vartheta} > 0$, therefore $\frac{\partial G}{\partial \vartheta} > 0$ and $\frac{\partial k^{ss}}{\partial \vartheta} > 0$.

When $\tau \in (0, \widehat{\tau})$:

$$\frac{\partial k^{ss}}{\partial \tau} = -\frac{\frac{\partial G}{\partial \lambda}\frac{\partial \lambda}{\partial \tau} + \frac{\partial G}{\partial \tau}}{\frac{\partial G}{\partial k}} = \frac{\frac{\partial G}{\partial \lambda}}{-\frac{\partial G}{\partial k}}\frac{\partial \lambda}{\partial \tau} + \frac{\frac{\partial G}{\partial \tau}}{-\frac{\partial G}{\partial k}} = \underbrace{\frac{\partial k}{\partial \lambda}}_{+}\underbrace{\frac{\partial \lambda}{\partial \tau}}_{+} + \frac{\frac{\partial G}{\partial \tau}}{-\frac{\partial G}{\partial k}} =$$
$$\underbrace{\frac{\partial k}{\partial \lambda}}_{+}\underbrace{\frac{\partial \lambda}{\partial \tau}}_{+} + \frac{\frac{\partial G}{\partial \tau}}{-(-1)} = \underbrace{\frac{\partial k}{\partial \lambda}}_{+}\underbrace{\frac{\partial \lambda}{\partial \tau}}_{+} + \frac{\partial G}{\partial \tau}$$

Note that:

$$\frac{\frac{\partial G}{\partial \tau}}{G(.)} = -\frac{1}{1-\alpha}\frac{\frac{\partial\left[1-\tau-g(l_p)\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p),\xi\right)\right]}{\partial \tau}}{1-\tau-g(l_p)\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p), \xi\right)} + \frac{\frac{\partial\left[\left(1-\frac{\vartheta\tau}{\lambda}\right)(1-l_p)\right]}{\partial \tau}}{\left(1-\frac{\vartheta\tau}{\lambda}\right)(1-l_p)} =$$
$$-\frac{1}{1-\alpha}\frac{g(l_p)\varphi'_{l_g}\left(\frac{\vartheta\tau}{\lambda}(1-l_p), \xi\right)}{1-\tau-g(l_p)\varphi\left(\frac{\vartheta\tau}{\lambda}(1-l_p), \xi\right)} + \frac{\frac{\partial[(1-l_p-l_g)]}{\partial \tau}}{1-l_p-l_g}$$

We have shown that for $\tau \in (0, \widehat{\tau})$ $\frac{\partial[(1-l_p-l_g)]}{\partial \tau} > 0$, therefore $\frac{\partial G}{\partial \tau} > 0$ and $\frac{\partial k^{ss}}{\partial \tau} > 0$.

Proof of Lemma 9

Using the fact that at equilibrium $\widetilde{i}_p = i_p$ and $\widetilde{y} = y$, it follows from (24) that:

$$\frac{w(t)l}{y(t)} = \frac{\theta(i_p)}{\eta(i_p)} \frac{l}{l_p} \frac{g(l_p)}{(1-g(l_p))} \Leftrightarrow \lambda = \frac{\theta(i_p)}{\eta(i_p)} \frac{l}{l_p} \frac{g(l_p)}{(1-g(l_p))} \tag{65}$$

where $l \equiv_{i_p}^{1} \eta(i)di$ and $l_p \equiv_0^{i_p} \theta(i)di$.

$$\frac{\partial \left[\frac{\theta(i_p)}{\eta(i_p)} \frac{l}{l_p} \frac{g(l_p)}{(1-g(l_p))}\right]}{\partial i_p} = \left[\frac{\theta(i_p)}{\eta(i_p)} \frac{l}{l_p} \frac{g(l_p)}{(1-g(l_p))}\right] \frac{\partial \ln \left[\frac{\theta(i_p)}{\eta(i_p)} \frac{l}{l_p} \frac{g(l_p)}{(1-g(l_p))}\right]}{\partial i_p} =$$

$$\underbrace{\left[\frac{\theta(i_p)}{\eta(i_p)} \frac{l}{l_p} \frac{g(l_p)}{(1-g(l_p))}\right]}_{\lambda} \left[\frac{\frac{\partial\left(\frac{\theta(i_p)}{\eta(i_p)}\right)}{\partial i}}{\frac{\theta(i_p)}{\eta(i_p)}} - \frac{\eta(i_p)}{l} - \frac{\theta(i_p)}{l_p} + \frac{g'(l_p)\theta(i_p)}{g(l_p)} + \frac{g'(l_p)\theta(i_p)}{(1-g(l_p))}\right]$$

where in the derivatives we have used the definitions of l and l_p. Using equation (65), it follows that:

$$\frac{\partial \left[\frac{\theta(i_p)}{\eta(i_p)} \frac{l}{l_p} \frac{g(l_p)}{(1-g(l_p))}\right]}{\partial i_p} =$$

$$\lambda \left[\frac{\frac{\partial\left(\frac{\theta(i_p)}{\eta(i_p)}\right)}{\partial i}}{\frac{\theta(i_p)}{\eta(i_p)}} - \frac{\theta(i_p)}{\lambda l_p} \frac{g(l_p)}{(1-g(l_p))} - \frac{\theta(i_p)}{l_p} + \frac{g'(l_p)\theta(i_p)}{g(l_p)} + \frac{g'(l_p)\theta(i_p)}{(1-g(l_p))}\right] =$$

$$\lambda \left[\frac{\frac{\partial\left(\frac{\theta(i_p)}{\eta(i_p)}\right)}{\partial i}}{\frac{\theta(i_p)}{\eta(i_p)}} - \theta(i_p) \left[\frac{(g(l_p))^2 \left[\frac{1-\lambda}{\lambda}\right] + g(l_p) - g'(l_p) l_p}{(1-g(l_p)) g(l_p) l_p}\right]\right] < 0$$

where in the above inequality, we have used the assumption that $\frac{\eta(i)}{\theta(i)}$ is increasing (therefore, $\frac{\theta(i)}{\eta(i)}$ is strictly decreasing), and the assumption that $g(.)$ is concave and $g(0) = 0$. It follows from the Taylor Theorem, the assumption that $g(.)$ is concave and from $g(0) = 0$ that:

$$\begin{aligned} g(0) &= g(l_p) - g'(l_p) l_p + g"(\xi l_p)(l_p)^2 < g(l_p) - g'(l_p) l_p \Rightarrow \\ g(l_p) &> g(0) + g'(l_p) l_p = g'(l_p) l_p \end{aligned}$$

The proposition is a result from equation (65) and the Implicit Function Theorem:

$$
\begin{aligned}
\frac{\partial i_p}{\partial \lambda} &= \frac{1}{\frac{\partial\left[\frac{\theta(i_p)}{\eta(i_p)}\frac{l}{l_p}\frac{g(l_p)}{(1-g(l_p))}\right]}{\partial i_p}} < 0 \\
\frac{\partial l_p}{\partial \lambda} &= \frac{\partial l_p}{\partial i_p}\frac{\partial i_p}{\partial \lambda} = \theta(i_p)\frac{\partial i_p}{\partial \lambda} < 0 \\
\frac{\partial l}{\partial \lambda} &= \frac{\partial l}{\partial i_p}\frac{\partial i_p}{\partial \lambda} = -\eta(i_p)\frac{\partial i_p}{\partial \lambda} > 0
\end{aligned}
$$

Profs Lemmas 10 and 11, and proposition 12:

The proof of lemma **10** is exactly the same as lemma **3.**

It follows from (25), and the fact that at equilibrium $\widetilde{l}_p = l_p$ and $\widetilde{y} = y$, that:

$$\frac{w(t)hl}{y} = \frac{g'_{l_p}(l_p)(1-l_p)}{[1-g(l_p)]} \Leftrightarrow \lambda = \phi(l_p)$$

which is exactly the same equation as (35). From this point, the proof of lemma **11** is exactly the same as lemma **4.**

Finally, solving the model it follows that:

$$\frac{\overline{c}}{\Gamma_a h^{1-\alpha-\beta} z^{\beta} k^{\alpha}} = \left(\frac{1-\alpha-\beta}{\lambda}\right)^{1-\alpha-\beta}\left(\frac{1-\alpha-\lambda}{\beta}\right)^{1-\beta}\left(1-l_p(\lambda)\right)^{1-\alpha-\beta} \tag{66}$$

From this point, the proof of proposition **12** is exactly the same as proposition **5.**

Proof of Lemma 13

Using the fact that at equilibrium $\widetilde{i}_p = i_p$ and $\widetilde{y} = y$ and $l_p + l = 1$, it follows from (33) that:

$$\lambda = \frac{w\left[l + l_p\varepsilon\right]}{y} = \frac{w\left[1 - l_p(1-\varepsilon)\right]}{y} = \frac{g'_{l_p}(l_p)\left[1 - l_p(1-\varepsilon)\right]}{\left[1-g(l_p)\right](1-\varepsilon)} \tag{67}$$

$$\frac{\partial\left[\frac{g'_{l_p}(l_p)[1-l_p(1-\varepsilon)]}{[1-g(l_p)](1-\varepsilon)}\right]}{\partial l_p} = \left[\frac{g'_{l_p}(l_p)\left[1-l_p\left(1-\varepsilon\right)\right]}{\left[1-g(l_p)\right]\left(1-\varepsilon\right)}\right]\frac{\partial\ln\left[\frac{g'_{l_p}(l_p)[1-l_p(1-\varepsilon)]}{[1-g(l_p)](1-\varepsilon)}\right]}{\partial l_p} =$$

$$\frac{g'_{l_p}(l_p)\left[1-l_p\left(1-\varepsilon\right)\right]}{\left[1-g(l_p)\right]\left(1-\varepsilon\right)}\left[\frac{g"_{l_p}(l_p)}{g'_{l_p}(l_p)}-\frac{(1-\varepsilon)}{\left[1-l_p\left(1-\varepsilon\right)\right]}+\frac{g'_{l_p}(l_p)}{\left[1-g(l_p)\right]}\right] =$$

$$\lambda\left[\frac{g"_{l_p}(l_p)}{g'_{l_p}(l_p)}-\frac{g'_{l_p}(l_p)}{\left[1-g(l_p)\right]\lambda}+\frac{g'_{l_p}(l_p)}{\left[1-g(l_p)\right]}\right] =$$

$$\lambda\left[\frac{g"_{l_p}(l_p)}{g'_{l_p}(l_p)}-\frac{g'_{l_p}(l_p)}{\left[1-g(l_p)\right]\lambda}\frac{1-\lambda}{\lambda}\right] < 0$$

where in the second equality we used equation (67) and in the final inequality we took into account the assumption that $g(.)$ is a concave function. Thus, applying Implicit Function Theorem to equation (67), we get:

$$\frac{\partial l_p}{\partial\lambda} = \frac{1}{\frac{\partial\left[\frac{g'_{l_p}(l_p)[1-l_p(1-\varepsilon)]}{[1-g(l_p)](1-\varepsilon)}\right]}{\partial l_p}} < 0$$