Social norms and economic growth in a model with labor and capital income tax evasion*

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Abstract

Empirical evidence suggests that low-income countries are characterized by high levels of labor and capital income tax evasion while the opposite is true for high-income countries. This paper proposes a model to study the relationship between economic growth and both types of income tax evasion. We show that the existence of a social norm towards tax compliance generates a complementarity between capital and labor income tax evasion which explains the decline of both the share of evaders in the population and the amount of tax evasion when countries accumulate capital. The model predicts that the level of tax morale is positively correlated with both types of income tax evasion and the level of income per capita, consistent with recent empirical evidence. Finally, a higher tax rate increases the share of evaders in the population and aggregate tax evasion.

Keywords: income tax evasion, social norms, economic growth

JEL Classification: D11, D91, Z13, H26

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1 Introduction

Tax evasion not only harms the possibility of governments to obtain public revenues and, thus, to finance public programs; tax evasion is also negatively related to economic performance. In a recent survey, Vasilopoulou and Thomakos (2017) provide evidence about a strong negative relationship between economic growth and tax evasion. Similarly, Schneider et al. (2011) and Crane and Nourzad (1986) find that the relative size of the shadow economy (and tax evasion) have decreased over time for a wide sample of countries whereas GDP per capita has increased. Besley and Persson (2014) and Gordon and Lee (2005) show that for similar tax rates high-income countries obtain much higher levels of tax revenue than low-income countries, suggesting that tax evasion is a severer problem in low-income countries. Moreover, empirical findings suggest that tax evasion is a widespread problem, especially in developing countries which show high levels of both labor and capital income tax evasion (e.g., Crivelli et al. (2016)).

The aim of this article is to analyze the channels through which tax evasion affects economic development and viceversa. We set up a general dynamic equilibrium model with production that considers both capital and labor income tax evasion. Our framework integrates the well-established Allingham-Sandmo-Yitzhaki setting into the standard overlapping generations model with two periods and a neoclassical technology. Moreover, agents face a nonpecuniary cost of evading taxes which depends on the tax-compliance behavior of other citizens (i.e., tax morale).

Taxpayers can conceal labor and/or capital income when paying taxes and may be audited by the government. When being detected, agents are fined. They also face a nonpecuniary cost which is formalized as a social normal towards tax compliance (tax morale). The strength of the norm depends positively on the fraction of evaders in the economy. Individuals choose optimal levels of labor and capital income tax evasion as well as savings. The dynamics of the economy are determined by aggregate savings. Along the transition, per capita capital grows, thereby generating a reduction in the rate of return and, hence, in the incentive to evade capital income taxes. A smaller number of agents evading capital income strengthens the social norm. This reduces the number of evaders of both capital and labor income which, in turn, increases the level of tax morale even more. Thus, a complementarity of both types of evasion emerges. The economic growth process is, hence, characterized by a reduction of tax evaders. The model also predicts that higher tax rates generate lower levels of tax evasion in the steady state and along the transition.

The social norm towards tax compliance is a key element in our model. Empirical studies emphasize the importance of taxpayers' moral values and attitudes for tax evasion (see, e.g., Luttmer and Singhal (2014)). This literature finds that the decision to evade taxes is not only shaped by pecuniary factors but also by morale and social considerations which are, in turn, determined by other citizens' behaviors. Individuals perceive evading taxes as a less dishonest behavior the more prevalent they expect it to be (see, e.g., Frey and Torgler (2007)). Thus, tax morale is modeled as a social norm towards tax compliance, implying an extra cost for evaders besides the fines they have to pay when being detected.

Our main contribution is to document the existence of a complementarity between tax evasion on different sources of income which emerges from the existence of a social norm. Whereas some paper focus on labor income tax evasion (see, for instance, Varvarigos (2017)) and others on capital income tax evasion (see Bethencourt and Kunze (2019)), this paper model both types of tax evasion simultaneously and characterizes the interdependence that exists between them. We show that the existence of a social norm towards tax compliance, which depends on the share of capital and labor income tax evaders, generates a complementarity of both types of evasion in the economy. A decrease in the share of evaders for one of these types of incomes strengths the tax morale, which in turn reduces incentives to evade both capital and labor income taxes. This complementarity accounts for the decline of the share of evaders in the economy and aggregate evasion when countries accumulate capital throughout the transition, which is consistent with empirical evidence suggesting that labor and capital income tax evasion are interrelated (see Besley and Persson (2014) and Crivelli et al. (2016)) and that such a complementarity is closely linked to a social norm towards tax compliance (Alm and Martinez-Vazquez, 2003).

Our second contribution is to show that this novel mechanism based on the complementarity of different types of evasion and its evolution along the transition helps us to understand the issues surrounding the relationship between tax evasion and economic development. Specifically, our model generates several results which are supported by empirical evidence. First, the model shows that there exists a positive relationship between labor and capital income tax evasion (see Besley and Persson (2014), Alm (2014), Crivelli et al. (2016) and Cobham and Janský (2018)). Second, the model predicts a negative relationship between per capita income and tax evasion (see Easterly and Rebelo (1993b), Easterly and Rebelo (1993a) and Gordon and Li (2009)). Third, from the neoclassical setting we also obtain that the share of capital and labor income tax evasion decreases insofar countries grow and accumulate capital (Schneider et al. (2011) and Crane and Nourzad (1986)).

Our third contribution is to show the existence of a new channel through which capital income tax evasion may affect economic growth. This is a new theoretical result in the literature on economic growth and tax evasion. In the canonical 2-OLG model with young and old individuals (Acemoglu, 2009) only taxes on labor income affect savings. Hence, capital taxation and so, capital income tax evasion would not affect capital accumulation directly (see Bethencourt and Kunze (2019)). In this paper, however, the complementarity between labor and capital income tax evasion implies that a higher level of tax morale (due to a lower share of capital income tax evaders) reduces the share of labor income evaders which, in turn, affects aggregate savings and thus capital accumulation.

Our fourth contribution is to show that there is a positive relationship between tax morale, both capital and labor income tax compliance, and per capita income, which is supported by empirical evidence (e.g., Torgler (2003)). In addition, the facts that countries with low quality institutions display higher levels of tax evasion (Torgler and Schneider, 2007) and that these countries are typically poor ones (Acemoglu et al., 2005), support the finding that developed countries show high degrees of tax morale, high quality institutions and low levels of tax evasion.[1]

Our paper is connected to a branch of literature which highlights the interactions between social norms of compliance and institutions (see Acemoglu and Jackson (2017), Besley and Persson (2014) and Benabou and Tirole (2011)). The existence of the complementarity mechanism between capital and labor income tax evasion amplifies the impact of any institutional reform devoted to reduce tax evasion: the direct effect of the institutional change will not only affect the targeted source of tax evasion but also other types of evasion through changes in the social norm.

A large number of papers has studied the relationship between tax evasion and economic growth considering dynamic general equilibrium models (Levaggi and Menoncin

[1]Similarly, Alm and Martinez-Vazquez (2003) state that tax evasion is often widespread in developing countries and that this is related to the level of tax morale. Empirical findings allow them to conclude that the analysis of the impact of social norms on compliance behavior is a key factor for understanding tax evasion in low-income countries.

(2013), Cerqueti and Coppier (2011), Dzhumashev and Gahramanov (2011) or Chen (2003)).[2] The focus of these studies, however, is on the long run. By contrast, the present study also considers the dynamics of tax evasion along the economic growth process. The closest papers to ours are Bethencourt and Kunze (2019) and Varvarigos (2017). Both papers consider a social norm towards tax compliance in a 2-OLG model and study how tax evasion evolves along the transition. However, whereas the first one does not consider labor income tax evasion, the second one ignores capital income tax evasion.

The reminder is structured as follows. In section 2 we present the economic setting and derive the main results of the paper. In section 3 we outline a version of the model with a productive public good. In section 4 we present several extensions and robustness checks related to some assumptions of our theory. Finally, section 5 concludes. The paper includes a technical appendix will all proofs.

2 The Model

We present a standard two-periods overlapping generations model with tax morale along the lines of Gordon (1989). Each young individual has one offspring so that the size of each generation remains constant and is, thus, normalized to one. We abstract from labor supply decisions, implying that agents own one unit of labor time in the first period of their life (the young adult age) and consume the return to their savings in the second period (the mature age). Markets are competitive. Individuals may evade labor and capital income taxes collected by the government.

[2] See also Dzhumashev (2014) and Célimène et al. (2016) who analyze the relationship between corruption, tax evasion and economic development.

2.1 Firms

We assume the existence of a large number of identical firms which act under perfect competition. Each firm uses labor L_t and capital K_t to produce the private good with a Cobb–Douglas production function:

$$Y_t = AK_t^{\alpha} L_t^{1-\alpha}, \tag{1}$$

and $0 < alpha < 1$ represents the capital share.

Under perfect competition, profit maximization by each firm implies that, in equilibrium, marginal products of factors equal their prices:

$$w_t = (1-\alpha)AK_t^{\alpha} L_t^{-\alpha} = (1-\alpha)Ak_t^{\alpha} \tag{2}$$

$$r_t = \alpha AK_t^{\alpha-1} L_t^{1-\alpha} = \alpha Ak_t^{\alpha-1} \tag{3}$$

where $k_t = K_t/L_t$.

2.2 Consumers

In the first period of life, the young adult age, individuals work and receive the wage w_t. However, they have to pay taxes and may evade a fraction $x_t \in [0,1]$ of their labor income. Taxes are calculated as a constant share τ of individuals' declared income. In line with the literature (see, e.g., Traxler (2010)), we assume that tax officers do not have information about agents' incomes (labor and capital income). Specifically, we assume that individuals' income is not publicly observable and thus private information for each individual. Therefore, fiscal authorities audit taxpayers. With probability q labor income tax evasion gets detected and so, evaders have to pay the owed (full) taxes plus a fine. The fine is calculated as a proportion γ of evaded taxes. With probability $1-q$ labor income tax evasion remains undetected and the taxpayer only pays the declared taxes. Labor income is spent on savings s_t and consumption c_t. Therefore, consumption levels in the first period for both states, getting detected (d-state) and remaining undetected (u-state), are as follows

$$c_t^d = w_t(1-\tau-\gamma\tau x_t) - s_t \tag{4}$$

$$c_t^u = w_t(1-\tau+\tau x_t) - s_t \tag{5}$$

When mature, agents consume the return to their savings. They have to pay a proportional tax τ on the realized capital income. However, they have the possibility to evade taxes. Let $e_{t+1} \in [0,1]$ be the share of evaded taxes. With probability p capital income tax evasion gets detected[3]. Then, as in the case of labor income, the taxpayer has to pay both the full tax that is owed and a fine. The fine is also calculated as a proportion γ of the evaded taxes. With probability $1-p$ capital income tax evasion remains undetected and the taxpayer only pays the declared taxes. Hence, consumption levels in the second period for both cases, detected and remaining undetected, are as follows

$$d_{t+1}^d = R_{t+1}^d s_t \tag{6}$$

$$d_{t+1}^u = R_{t+1}^u s_t \tag{7}$$

[3]We are assuming that the probability of detecting labor income tax evasion may be different to the one of detecting capital income tax evasion. Whereas pensions and wages are automatically reported to the tax administration, capital income is much more difficult to control since it may be earned through offshore financial institution, tax heavens, cross-border investments and sophisticated forms of business. The literature, in fact, suggests that the first probability is larger than the second one, i.e. $q > p$ (see Alstadsæter et al. (2017) for an overview).

where the capital depreciation rate is denoted by $\delta \in [0,1]$, and $R^d_{t+1} = 1-\delta + r_{t+1}(1-\tau - \gamma\tau e_{t+1})$ and $R^u_{t+1} = 1-\delta + r_{t+1}(1-\tau + \tau e_{t+1})$ represent the disposable (after-tax) private returns on capital upon detection and when remaining undetected respectively. Finally, we assume that both young and mature individuals enjoy a public good, g_t (financed by tax revenues).

Agents face nonpecuniary costs when evading taxes. As in Gordon (1989), these psychological costs are related to individuals' reputation and morality concerns[4]. In line with the existent literature, we assume that moral costs are a linear function of the fraction of concealed income. Moreover, moral costs are formalized as a social norm towards tax compliance (tax morale), where the strength of the norm is endogenously determined by the share of the population that adheres to it (Jackson et al. (2017) and Antoci et al. (2014)). The intuition is the following: the lower the number of evaders (the higher the number of individuals that adheres to the norm), the more agents' perceive the act of evading taxes as wrongdoing. Agents would find it harder to justify their own tax evasion, perceiving tax compliance as the right behaviour, which in turn encourages them to pay the owed taxes. Thus, individuals' preferences are not only affected by consumption decisions, but also depend on the 'moral costs' derived from the tax evasion decisions. Finally, note that since taxes from different sources of income are evaded in different time periods, this also has to be the case for the social norm and the corresponding moral cost that applies in each period[5]. Accordingly, a young agent evading labor income taxes has a moral cost which depends on the current share of evaders in society. However, when she becomes mature and decides evading capital income taxes in the second period, she

[4]Other approaches that extend the static Allingham-Sandmo-Yitzhaki setting consider nonpecuniary costs linked to stigmatization concerns or preferences for tax compliance (see Traxler (2010) for a detailed overview of the literature.

[5]An alternative approach is to assume that both labor and capital income tax evasion are affected by the same social norm. This approach yields very similar results to the ones we obtain in the paper.

will face moral costs that will depend on the future share of evaders. Therefore, we represent the lifetime utility function of an agent i with a degree of norm internalization θ_i born in period t as:

$$U_i(c_t^u, c_t^d, d_{t+1}^u, d_{t+1}^d, e_{t+1}) = (1-q)ln(c_t^u) + q ln(c_t^d) + v(g_t) - x_t(\theta_i + \mu(1-n_t))$$
$$+(1-p)\beta ln(d_{t+1}^u) + p\beta ln(d_{t+1}^d) + \beta v(g_{t+1}) - e_{t+1}(\theta_i + \mu(1-n_{t+1})) \quad (8)$$

where $0 < v' > 0$, $0 < \beta$ is the intertemporal discount factor, and expressions $x_t(\theta_i + \mu(1-n_t))$ and $e_{t+1}(\theta_i + \mu(1-n_{t+1}))$ represent the moral costs of capital and labor income tax evasion, respectively. Moral costs are affected by the individual degree of norm internalization $\theta_i \in [0, \bar{\theta}]$ which is distributed according to the function $F(\theta_i)$. Moreover, moral costs are also affected by the portion of evaders in the population, n_t, and the intrinsic reputation cost, $0 < \mu$. [6] In addition, it is assumed that moral costs depend positively and linearly on the amount of tax evasion (labor income tax evasion, x_t, and capital income tax evasion, e_{t+1}). Traxler (2010) and Gordon (1989) establish equivalent assumptions but in alternative static frameworks[7]. The main differences with respect to other studies in the literature are twofold: first, whereas most of papers in the literature model tax evasion in levels, we model tax evasion as a share of the levied taxes[8] and, second, we

[6]Note that even in the case $\mu = 0$ the main results of the paper would remain unchanged. The dynamic of the economy, however, would be fully shaped by the capital accumulation process

[7]The social norm-related and individual-specific elements of the moral cost function, θ_i, and $\mu(1-n_t)$ and $\mu(1-n_{t+1})$ respectively, are assumed to be separable. We have followed the standard approach of Gordon (1989) at this point. This implies that agents always have some moral concerns, even when $n = 1$. We have also considered the linearity assumption because of simplicity and because it is a standard assumption in the literature. However, our main results are robust to more complex formalizations like convex cost functions.

[8]We capture the idea that the size of the moral cost depends on the relative size of evaded taxes with respect to the levied tax that should be paid. Imagine two taxpayers with different income levels evading the same amount of taxes. Given the poorer agent is evading a larger percentage of taxes, this individual might experience a higher moral concern than the richer one who is evading a lower percentage. Other papers with a similar approach are Levaggi and Menoncin (2013) and Bosco and Mittone (1997).

consider two different types of evasion and the interrelationship between them.

The lifetime utility function can be rewritten in a more concise way, that is,

$$U_i(c_t^u, c_t^d, d_{t+1}^u, d_{t+1}^d, x_t, e_{t+1}) = E[U(c_t^u, c_t^d, d_{t+1}^u, d_{t+1}^d)] - C(x_t, e_{t+1}, n_t, n_{t+1}) \tag{9}$$

where $E[U(c_t^u, c_t^d, d_{t+1}^u, d_{t+1}^d)] = (1-q)ln(c_t^u) + q ln(c_t^d) + v(g_t) + (1-p)\beta ln(d_{t+1}^u) + p\beta ln(d_{t+1}^d) + \beta v(g_{t+1})$ and $C(x_t, e_{t+1}, n_t, n_{t+1}) = x_t(\theta_i + \mu(1-n_t)) + e_{t+1}(\theta_i + \mu(1-n_{t+1}))$ represent the pecuniary and the non-pecuniary (moral cost) components of the utility, respectively. Each agent maximizes utility (9), subject to eqs. (5), (4), (7) and (6), by choosing c_t^u, c_t^s, s_t, x_t, e_{t+1}, d_{t+1}^u and d_{t+1}^d, given n_t, n_{t+1} and prices. The first order conditions (FOCs) with respect to s_t, x_t and e_{t+1} for an interior solution are:

$$E[U(.)]'_s \equiv -\left[\frac{1-q}{c_t^u} + \frac{q}{c_t^d}\right] + \frac{\beta}{s_t} = 0 \tag{10}$$

$$E[U(.)]'_x \equiv \tau w_t \left[\frac{1-q}{c_t^u} - \gamma\frac{q}{c_t^d}\right] = \theta_i + \mu(1-n_t) \tag{11}$$

$$E[U(.)]'_e \equiv \beta\tau r_{t+1}\left[\frac{1-p}{R_{t+1}^u} - \gamma\frac{p}{R_{t+1}^d}\right] = \theta_i + \mu(1-n_t) \tag{12}$$

Note that the FOC with respect to s_t, eq. (10) is not affected by morality. The respective second order conditions (SOCs) are:

$$E[U(.)]''_s \equiv -\left[\frac{1-q}{(c_t^u)^2} + \frac{q}{(c_t^d)^2}\right] - \frac{\beta}{(s_t)^2} < 0 \tag{13}$$

$$E[U(.)]''_x \equiv -(\tau w_t)^2\left[\frac{1-q}{(c_t^u)^2} + \gamma\frac{q}{(c_t^d)^2}\right] < 0 \tag{14}$$

$$E[U(.)]''_e \equiv -\beta(\tau r_{t+1})^2\left[\frac{1-p}{(R_{t+1}^u)^2} + \gamma^2\frac{p}{(R_{t+1}^d)^2}\right] < 0 \tag{15}$$

The optimal share of labor income concealed, x_t^*, is characterized by eq. (11). Guided by the social norm, agents choose an optimal fraction of labor income tax evasion such that marginal moral costs of evading, $\theta_i + \mu(1-n_t)$, coincide with the marginal expected utility, $E[U(.)]'_x$. We need the evasion gamble better than fair in order to have an interior solution, that is,[9], i.e.

$$z(w_t) \equiv E[U(x_t = 0)]' = \frac{(1-q(1+\gamma))\tau(1+\beta)}{(1-\tau)} > 0 \tag{16}$$

From eqs. (11) and (14) we conclude that agents with $z(w_t) < \theta_i + \mu(1-n_t)$ prefer to not conceal any labor income. Therefore, the threshold

$$\hat{\theta}^w(n_t) \equiv z(w_t) - \mu(1-n_t) \tag{17}$$

allows us to characterize individuals' optimal decisions on labor income tax evasion $x_t^{*,i}$ for a given level of n_t.

$$x_t^{*,i} = \begin{cases} 0 & \text{for } \theta_i \geq \hat{\theta}^w(n_t) \\ x_t^{*,i} & \text{for } \theta_i < \hat{\theta}^w(n_t) \end{cases} \tag{18}$$

Those individuals with $\theta_i < \hat{\theta}^w(n_t)$ decide to evading a positive amount of taxes, $x_t \leq 0$, whereas those with $\theta_i \geq \hat{\theta}^w(n_t)$ do not evade taxes and comply with the norm.

The optimal share of capital income concealed, e_{t+1}^*, is characterized by eq. (12). Guided by the social norm, agents choose an optimal fraction of evaded taxes such that marginal moral costs, $\theta_i + \mu(1-n_{t+1})$, coincides with the marginal expected utility, $E[U(.)]'_e$. The evasion gamble has cannot be fair in order to have an interior solution,

[9]We then have to assume that $\gamma < (1-q)/q$ or equivalently $0 < 1-q(1+\gamma)$. Otherwise, i.e. $0 > 1-q(1+\gamma)$, individuals would never evade taxes which is not realistic. Other papers in the tax evasion literature establish equivalent assumptions.

that is, we need

$$z(r_{t+1}) \equiv E[U(0)]' = \frac{(1-p(1+\gamma))\beta\tau r_{t+1}}{1-\delta+(1-\tau)r_{t+1}} > 0 \tag{19}$$

Hence, we need to guarantee that $1-p(1+\gamma)>0$, which is the same assumption we need for having $x_t^* > 0$. However, while $z(w_t)$ (eq. 16) is independent of w_t and k_t since it is constant along time, $z(r_{t+1})$ (eq. 19) depends on r_{t+1} and so, on the per capita capital of the economy. As in the case of labor income tax evasion, taxpayers with $z(r_{t+1}) < \theta_i + \mu(1-n_{t+1})$ are not concealing any income. Therefore, the threshold

$$\hat{\theta}(n_{t+1}, r_{t+1}) \equiv z(r_{t+1}) - \mu(1-n_t) \tag{20}$$

allows us to characterize individuals' optimal decisions on capital income tax evasion $e_{t+1}^{*,i}$, for given levels of r_{t+1} and n_{t+1}:

$$e_{t+1}^{*,i} = \begin{cases} 0 & \text{for } \theta_i \geq \hat{\theta}(n_{t+1}, r_{t+1}) \\ e_{t+1}^{*,i} & \text{for } \theta_i < \hat{\theta}(n_{t+1}, r_{t+1}) \end{cases} \tag{21}$$

Taxpayers with relatively high moral concerns, $\hat{\theta}(n_{t+1}, r_{t+1}) > \theta_i$, evade a positive amount of capital income taxes, $e_{t+1}^{*,i} \leq 0$, whereas taxpayers with low moral concerns, $\hat{\theta}(n_{t+1}, r_{t+1}) < \theta_i$, decide to comply with the norm.

The fractions of evaded labor and capital income decrease as tax enforcement becomes more severe, that is, $dx_t^*/dq < 0$, $dx_t^*/d\gamma < 0$ and $de_{t+1}^*/dp < 0$, $de_{t+1}^*/d\gamma < 0$ for those individuals with $\theta_i < \hat{\theta}^w$ and $\theta_i < \hat{\theta}$ respectively. We obtain these results by implicitly differentiating the first order conditions (11) and (12). Moreover, both $\hat{\theta}^w$ and $\hat{\theta}$ fall, which implies that the number of agents evading zero taxes, $x_t^* = 0$ and $e_{t+1}^* = 0$ rises. Hence, both labor and capital income tax evasion decrease. The following proposition describes the how changes in r_{t+1} and τ affect to capital income tax evasion.

Proposition 1 *There is some* $\tilde{\theta}(n_{t+1}, r_{t+1}) < \hat{\theta}(n_{t+1}, r_{t+1})$ *such that* $\partial e^{*,i}_{t+1}/\partial r_{t+1} > 0$, $\partial e^{*,i}_{t+1}/\partial \tau > 0$ *if* $\theta_i > \tilde{\theta}(n_{t+1}, r_{t+1})$ *and* $\partial e^{*,i}_{t+1}/\partial r_{t+1} \leq 0$, $\partial e^{*,i}_{t+1}/\partial \tau \leq 0$ *if* $\theta_i < \tilde{\theta}(n_{t+1}, r_{t+1})$ *for all* n_{t+1} *and* k_{t+1}.

Proof: *See Appendix.*

When the tax rate increases only the more honest agents decide to evade more taxes, whereas the dishonest ones prefer to evade less taxes. This implies that the degree of norm internalization plays a key role in shaping the impact of a shift in the tax rate.[10] The intuition is as follows: first, there is a negative income effect since the rise in the tax rate reduces agents' disposable income. This makes them take less risky decisions, which implies a reduction in their tax evasion activity. Second, the increase in the tax rate also rises the marginal costs and the marginal benefits of tax evasion (the pecuniary cost, derived from fines, plus the non-pecuniary cost, due to morality concerns). However, marginal costs increase less than marginal benefits. The reason is that the non-pecuniary part of the cost (tax morale cost) is assumed to be dependent on the amount of concealed income rather than the amount of evaded taxes. Thus, whereas the pecuniary cost increases proportionally to the tax rate (fines are assumed to be determined as a proportion of the amount of evaded taxes) the non-pecuniary cost remains constant. As a result, (aggregated) marginal costs increase less than marginal benefits, generating a substitution effect which encourages taxpayers to evade more taxes. We show that the overall effect (resulting from adding income and substitution effect) depends on taxpayers' degree of morality: for relatively honest taxpayers ($\tilde{\theta} < \theta_i$), the substitution effect

[10] Traxler (2010), Gordon (1989) and Bethencourt and Kunze (2019) obtain similar results. Whereas the first two papers consider a static framework and tax evasion in levels, however, the last one considers a dynamic framework and the share of evaded taxes.

prevails, producing a rise in the amount of evaded taxes as the tax rate increases, however, for highly dishonest taxpayers ($\tilde{\theta} > \theta_i$), the negative income effect prevails and tax evasion decreases. Therefore, when the tax rate increases, only the more dishonest taxpayers act in accordance with the standard portfolio theory and reduce the risky asset holdings. By contrast, the more honest agents decide to evade more taxes.

Moreover, we prove that a rise in the rate of return generates similar effects to those stemming from a higher tax rate. More precisely, an increase in the rate of return increases both the marginal benefits and the marginal costs of evading taxes. However, since the non-pecuniary part of the marginal costs (tax morale) does not depend on the amount of evaded taxes but on the amount of concealed income, this remains unchanged, increasing only the pecuniary part (fines upon detection). Consequently, marginal benefits increase more than marginal costs, thereby producing a substitution effect which encourages taxpayers to evade more taxes. On the other hand, the increase in the rate of return implies a positive income effect since it increase capital income. Consequently, taxpayers evade more taxes, however, since the income elasticity of capital evasion demand is less than unity, the portion of evaded taxes reduces. Therefore, the sign of the total effect relies upon the relative size of a positive income effect which encourages tax evasion and a substitution effect which discourages it. We show that for the more dishonest taxpayers ($\tilde{\theta} > \theta_i$) the substitution effects dominates and tax evasion increases, while the opposite is true for the more honest taxpayers.

Finally, if we differentiate (17) with respect to r_{t+1} and τ, we obtain

$$\frac{\partial\hat{\theta}(n_{t+1}, r_{t+1})}{\partial r_{t+1}} > 0 \quad \text{and} \quad \frac{\partial\hat{\theta}(n_{t+1}, r_{t+1})}{\partial \tau} > 0. \tag{22}$$

This implies that increases in both the tax rate and the rate of return generate an emergence of new evaders which, in turn, tends to rise the aggregate level of tax evasion.

Similarly, the following proposition describes the effects of a change in τ and w_t on labor income tax evasion:

Proposition 2 *$\partial x_t^{*,i}/\partial w_t = 0$ and there exists some $\underline{\beta} > 0$ such that $\partial x_t^{*,i}/\partial \tau > 0$ if $\underline{\beta} < \beta$ for all n_t and k_t.*

Proof: *See Appendix.*

A change in the tax rate produces a much more sophisticated effect on labor income tax evasion than on capital income tax evasion: as explained above, there is a negative income effect discouraging evasion and a substitution effect encouraging it. However, this negative income effect in the first stage also produces a decrease in savings, which dampens the reduction in tax evasion due to a lower disposable income. Note that if individuals have a low discount factor of future consumption, i.e. $\underline{\beta} < \beta$, and consumption smoothing is relevant enough, then savings decline considerably. In this case, the substitution effect prevails and labor income tax evasion increases with higher tax rates. However, while e_{t+1} is influenced by shifts in the interest rate (capital income), x_t is unaffected by changes in w_t, i.e., $dx_t^*/dw_t = 0$. The intuition is as follows: first, both the marginal costs of evasion as well as the marginal benefits increase. Nevertheless, given that the moral cost of evading taxes is affected by the portion of concealed income, the rise in marginal costs is larger than the rise marginal benefits, producing a substitution effect that encourages taxpayers to evade more taxes. Second, there exists a positive income effect which rises the size of tax evasion. Nevertheless, since the income elasticity of labor income tax evasion demand is below the unity, the portion of tax evasion on total income remain unchanged.

Analogously to $\hat{\theta}(n_{t+1}, r_{t+1})$, differentiation of (17) yields $\frac{\partial \hat{\theta}^w(n_t)}{\partial \tau} > 0$. This implies that the emergence of new labor income tax evaders increases the share of aggregate evasion. Evaders increase the share of evaded labor income taxes in the first period when the tax rate rises.

Empirical findings support the predictions derived in propositions (1) and (2): there is a positive correlation between tax rates and taxpayers' decisions of evading taxes, there exist individuals that never evade taxes even when this is the profitable choice and, finally, it is observed that the taxpayers' decisions about evading taxes are interdependent (see, among others, Traxler (2010)).

Finally, the effect of a change in the aggregate share of evaders n_t and the level of per capita capital k_t on individual's savings are established in the following proposition:

Proposition 3 $\partial s_t^{*,i}/\partial k_t > 0$ *and there exists some* $\underline{\beta} > 0$ *such that* $\partial s_t^{*,i}/\partial n_t > 0$ *if* $\underline{\beta} < \beta$.
Proof: *See Appendix.*

The effect of an increase in per capita capital on individual savings is the standard one: a higher stock of capital increases wages which in turn, raises savings. However, a higher share of evaders also has a positive effect on individual's savings by reducing the moral cost of tax evasion, which, in turn, increases individuals' disposable income.

2.3 Steady State and Dynamics

We define the equilibrium and the steady state. First, the economy is in equilibrium when agents maximize their objective functions and markets clear. Formally, given a positive starting level of the portion of evaders in population, $n_0 > 0$, a positive starting level of the capital stock, $k_0 = s_{-1} > 0$, and a fiscal policy (defined by the parameters p, q,

τ and γ), an intertemporal competitive equilibrium is an allocation

$$\{c_t^u, c_t^d, d_t^u, d_t^d, n_t, s_t, k_t, e_t, x_t; r_t, w_t\}_{t=0}^{\infty}$$

such that $\forall t$ the solution of consumers' maximization problem is given by c_t^u, c_t^d, d_t^u, d_t^d, s_t, e_t, x_t, the solution of the firm's maximization problem is given by L_t, k_t, capital and labor markets clear, i.e. $k_t = \bar{s}_{t-1}$ and $L_t = 1$, and the government devotes the entire tax collection, T_t, to provide the public good, i.e.,

$$g_t = T_t = \tau\left([1-(1-p(1+\gamma))\bar{e}_t]r_t\bar{s}_{t-1} + [1-(1-q(1+\gamma))\bar{x}_t]w_t\right) \tag{23}$$

where $\bar{x}_t$ and $\bar{e}_t$ represent aggregate shares of labor and capital income tax evasion, respectively, and $\bar{s}_{t-1}$ denotes aggregate savings in the economy.

The dynamic system of this economy consists of the equation that characterizes the capital accumulation process and the equation that describes the evolution of the portion of evaders in the population. The amount of capital at period $t+1$ is determined by the aggregate amount of savings of period t, that is,

$$k_{t+1} = \bar{s}_t = \int_0^{\bar{\theta}} s_t^{*,i} f(\theta_i) d\theta_i \tag{24}$$

with $f(\theta_i) = F' > 0$. Obviously, savings differ among individuals: whereas agents with relatively high levels of tax morale, i.e. $\bar{\theta} \geq \theta_i \geq \hat{\theta}^w$, do not evade labor income taxes, for those with relatively low levels of tax morale, i.e. $\theta_i < \hat{\theta}^w$, the share of evasion will be affected by their individual degree of morality. The fact that the level of tax morale depends on the total number of evaders in society, n_t, implies that aggregate savings will also depend on n_t[11].

Individuals have the possibility to evade taxes with respect to both labor and capital income. Nevertheless, to obtain the number of evaders in period $t+1$, we need to compute the total number of young individuals that decide to evade labor income at time t plus the total number of mature agents that are evading capital income at time t. Thus,

$$n_{t+1} = \frac{F\left(\hat{\theta}^w(n_t)\right) + F\left(\hat{\theta}(n_t, r_t)\right)}{2} \tag{25}$$

where the share of evaders in period $t+1$ depends positively on the share of evaders in period t.

It is straight forward to show that $\hat{\theta}^w(n_t) > \hat{\theta}(n_t, r_t)$ if $p = q$. This would imply that all taxpayers that conceal capital income would also conceal labor income whereas the opposite is not true. Apparently, this result would run against the empirical evidence which suggests that the size of capital income tax evasion is larger than the size of labor income tax evasion. For example, Poterba (1987) and Sandmo (2012) argue that better opportunities to hide capital income are due to the relatively low probability of detection relative to other income sources. Thus, increasing sufficiently the parameter q relative to p would make the model consistent with the empirical findings.

The average share of labor income tax evasion in period t, $\bar{x}_t$, and the average share of capital income tax evasion in period t, $\bar{e}_t$, are respectively given by

$$\bar{x}_t = \int_0^{\bar{\theta}} x_t^{*,i} f(\theta_i) d\theta_i = \int_0^{\hat{\theta}^w(n_t)} x_t^{*,i} f(\theta_i) d\theta_i. \tag{26}$$

[11]As it is shown in proposition 3, an increase in n_t reduces the marginal cost of labor income tax evasion, thereby increasing the young evaders' disposable income and so their savings. Moreover, it also increases the threshold value $\tilde{\theta}^w$, which implies the emergence of new evaders and so, higher aggregate savings.

$$\bar{e}_t = \int_0^{\bar{\theta}} e_t^{*,i} f(\theta_i) d\theta_i = \int_0^{\hat{\theta}(n_t, r_t)} e_t^{*,i} f(\theta_i) d\theta_i. \tag{27}$$

The derivatives of $\bar{x}_t$ and $\bar{e}_{t+}$ with respect to k_t can be written as follows[12]:

$$\frac{\partial \bar{x}_t}{\partial k_t} = \left[\int_0^{\hat{\theta}^w(n_t)} \frac{\partial x_t^{*,i}}{\partial w_t} f(\theta_i) d\theta_i + x_t^{*,i}|_{\theta_i = \hat{\theta}^w(n_t)} \frac{\partial \hat{\theta}^w(n_t)}{\partial w_t} \right] \frac{\partial w_t}{\partial k_t} = 0 \tag{28}$$

$$\frac{\partial \bar{e}_t}{\partial k_t} = \left[\int_0^{\hat{\theta}(n_t, r_t)} \frac{\partial e_t^{*,i}}{\partial r_t} f(\theta_i) d\theta_i + e_t^{*,i}|_{\theta_i = \hat{\theta}(n_t, r_t)} \frac{\partial \hat{\theta}(n_t, r_t)}{\partial r_t} \right] \frac{\partial r_t}{\partial k_t} \tag{29}$$

Note that both derivatives can be decomposed into the sum of two elements: the first one captures the change in the aggregate share of evasion due to individuals' behavioral responses whereas the second one captures the shift in the portion of evaders in the population. Regarding the second summand, it is easy to see that $\partial \hat{\theta}^w / \partial w_t > 0$ and $\partial \hat{\theta} / \partial r_t > 0$, i.e., that both the fraction of labor income evaders and the fraction of capital income evaders increase. However, since marginal effects are equal to zero, i.e. ($x_{t+1}^{*,i}|_{\theta_i = \hat{\theta}(n_t, r_t)} = 0$ and $e_t^{*,i}|_{\theta_i = \hat{\theta}(n_t, r_t)} = 0$), both expressions vanish. The first summand is equal to zero in the case of labor income tax evasion as the share of concealed labor income is independent of the amount of labor income[13], i.e., $\frac{\partial x_t^{*,i}}{\partial w_t} = 0$. Hence, the share of evaded labor income does not depend on the capital level of the economy. In the case of capital income tax evasion, however, we can decompose the first summand into a positive effect for highly honest taxpayers ($\tilde{\theta}(n_t, r_t) < \theta_i$) and a negative effect for dishonest taxpayers ($\tilde{\theta}(n_t, r_t) > \theta_i$) as proposition 1 shows. Thus, the net effect critically depends on how types θ_i, are distributed, i.e. on $F(\theta_i)$. Consequently, the relationship between the aggregate portion of capital and labor income tax evasion and the level of capital in the economy is determined exclusively by the relationship between capital income tax evasion and the

[12] See the Leibniz rule for differentiation of definite integrals.
[13] See proposition 1.

level of per capita capital (eq. 29).

A steady state is an equilibrium such that $n_{t+1} = n_t = n_*$ and $k_{t+1} = k_t = k_*$. Let us focus first on the non-linear eq. (25). This equation has a plethora of plausible and possible solutions. The following proposition establishes conditions for the existence of an interior steady state.

Proposition 4 *A sufficient condition for the existence of an interior solution, $n_* \in (0,1)$, to (25) is given by*

$$\mu < \hat{\theta}^w(1); \hat{\theta}(k_*, 1) < \bar{\theta} \tag{30}$$

Proof: *See Appendix.*

where k_* is the steady state solution.

We show that the dynamic system of this economy (described by eqs. 24) and (25)) possesses a saddle steady state (n_*, k_*) when the following (sufficient) condition is satisfied[14]:

$$\frac{\mu \Phi_{k_t}(n_*, k_*)}{2(1-\Phi_{n_t})} \left(F'(\hat{\theta}^w(n_*, k_*)) + F'(\hat{\theta}(n_*, k_*)) \right) + \frac{\frac{\partial \hat{\theta}}{\partial r_t} \alpha (1-\alpha) A k_*^{\alpha-2}}{2} F'(\hat{\theta}(n_*, k_*)) > 1 \tag{31}$$

where constants $\Phi_{k_t}(n_*, k_*) > 0$ and $\Phi_{n_t}(n_*, k_*) > 0$ are defined in the appendix.

Depending on the functional form of the distribution function F, one or multiple equilibria may arise[15]. However, it has also been shown that if F is distributed uniformly and $\bar{\theta}$ is large enough, then the stability condition is satisfied and a unique steady state (n_*, k_*) emerges[16]. In this latter case, we can explicitly determine the sign of eq. (29).

[14] See the appendix.
[15] See Traxler (2010).
[16] See Gordon (1989).

More precisely, we get

$$sign\left(\frac{\partial \bar{e}_t}{\partial k_t}\right) = \tilde{m} + (\gamma - 1)R_t\mu(1-n_t) - \beta\tau\gamma r_t \tag{32}$$

with

$$\tilde{m} = \sqrt{(\beta\tau\gamma r_t - \mu(1-n_t)(\gamma+1)R_t)^2 + 4p\beta\tau\gamma r_t(\gamma+1)R_t\mu(1-n_t)}. \tag{33}$$

and $R_t = 1-\delta+(1-\tau)r_t$. Straightforward calculations show that

$$sign\left(\frac{\partial \bar{e}_t}{\partial k_t}\right) < 0 \quad \Leftrightarrow \quad \hat{\theta}(n_t, r_t) > 0. \tag{34}$$

The following proposition summarizes this finding:

Proposition 5 *Consider that θ_i is uniformly distributed on the interval $[0,\bar{\theta}]$. If the per capita capital increases, then the portion of capital income evasion decreases along the transition to the steady state, $\partial \bar{e}_t / \partial k_t < 0$.*

Proof: *See Appendix.*

In order to understand the above result, assume that the initial level of per capita capital in the economy is lower than the steady state level, i.e., $k_0 < k^*$. In this case, the relatively low level of capital implies that its marginal product (and therefore the rate of return) is large (eq. 3). Hence, there is a strong incentive for capital income tax evasion. Note that the threshold level determining the share of capital income tax evaders in the society (see eq. (22)) increases with the rate of return. Thus, the high rate or return generates a high share of capital income tax evaders. Contrary, if we look at the level of tax morale, a large share of capital income tax evaders results in a low moral cost of evading taxes as the strength of the social norm towards tax compliance decreases in the share of capital income evaders. This low moral cost renders evading capital income taxes but also labor income taxes more profitable. Hence, the initial state of the economy is characterized by a low level of per capita capital and a large share of evaders.

The high rate of return will also increase individuals' savings and investments which in turn generates a capital accumulation process. Insofar the economy accumulates capital, the rate of return decreases, thereby discouraging capital income tax evasion. As a result, the number of capital income tax evaders declines. Existing evaders, however, react differently according to their idiosyncratic level of morality or honesty (see proposition 1). Still, proposition 5 shows that if θ_i is uniformly distributed then the negative effect prevails and the aggregate share of capital income tax evasion decreases. However, these effects are reinforced by changes of the social norm. In our model, a reduction of capital income tax evaders increases the moral cost of evading taxes and so it reduces the incentive to evade taxes. However, the social norm also affects labor income taxpayers. The increase in the moral cost reduces their incentives to evade labor income taxes. This, in turn, implies that the share of labor income tax evaders also declines, thereby increasing the level of tax morale even more. Thus, the decrease in the share of labor income tax evaders reinforces the decline in the share of capital income evaders and so on. A complementarity of both types of tax evasion emerges. Moreover, the reduction of labor income tax evaders implies a higher share of individuals fulfilling their tax duties and so a decrease in savings. Since a fraction of young agents evades labor income taxes, the economy will end up in a steady state with a larger capital stock as compared to a situation in which young agents do not evade taxes. Therefore, the transition towards the steady state is characterized by increases in the level of tax morale and simultaneous reductions in both the share of capital and labor income tax evaders, which are reinforced by the complementarity between labor and capital income tax evasion.

Several insights emerging from proposition 5 are noteworthy. First, the complementarity between capital and labor income tax evasion accounts for both the decline of the shares of capital and labor income tax evaders in the economy and the amount of evasion along the transition towards the steady state. This result is consistent with empirical evidence showing that low income countries, characterized by large informal sectors intensive in unskilled labor (Gërxhani, 2004), not only show high levels of labor income tax evasion (see, for instance, Besley and Persson (2014), Alm (2014)) but also high levels of capital income tax evasion (see, among others, Cobham and Janský (2018) and Crivelli et al. (2016)). Second, the complementarity of different types of evasion and its evolution along the transition generate a negative a relationship between tax evasion and economic development. While richer countries have lower levels of capital and labor income tax evasion, poorer ones show high levels of both types of evasion. In this regard, Easterly and Rebelo (1993a), Easterly and Rebelo (1993b) and Gordon and Lee (2005) document that, for similar preferences and technologies and the same size of tax rates, developed countries have less tax evasion than developing ones. Moreover, from the neoclassical framework we also obtain that the share of capital and labor income tax evasion decreases insofar countries grow and accumulate capital. This is consistent with empirical findings by Schneider et al. (2011), who analyze the evolution of aggregate tax evasion for a sample of developed and developing countries over the 2000s, and Crane and Nourzad (1986) who develop a similar analysis for the US using data from 1947 to 1981. Third, the model also predicts that low-income economies are characterized by both a high portion of labor and capital income evaders in the population, a high level of tax evasion and a low degree of tax morale. This positive relation between tax morale and per capita income is supported by empirical findings (see Torgler and Schneider (2007)).

The complementarity between capital and labor income tax evasion generates a new channel through which capital income tax evasion may affect economic growth. This is a new theoretical result in the literature on economic growth and tax evasion. While it is well-known that in the canonical standard 2-OLG model with young and old agents only taxes on labor income affect savings (Acemoglu (2009)), capital taxation and so, capital income tax evasion, would not affect capital accumulation directly (see Bethencourt and Kunze (2019)). However, this is not true in our model. Insofar countries grow during the transition, the rate of return decreases and thus also the share of capital income evaders. The reduction of capital income evaders increases the moral cost of evading taxes and thereby reduces the incentive to evade both capital and labor income taxes. Consequently, the share of labor income tax evaders also declines, which in turn implies a higher share of individuals fulfilling their tax duties and so a decline in capital accumulation.

Note that Chen (2003) finds similar results to ours in a model without tax morale. He proposes a standard AK growth model with public capital in which the representative household does not face a moral cost but rather a pecuniary cost of tax evasion. When the government decides the optimal tax rate, he shows that the economic growth rate is smaller and the optimal tax rate is larger in an economy with tax evasion than in an otherwise identical economy without tax evasion. The reason is that tax evasion reduces tax collection and so, the amount of the productive good. Moreover, he also proves that if the cost of tax enforcement is not too high and the degree of the government externality is sufficiently large, then enforcement policies would generate both low levels of tax evasion and high rates of economic growth.

The derivatives of $\bar{x}_t$ and $\bar{e}_{t+1}$ with respect to τ are given by

$$\frac{\partial \bar{x}_t}{\partial \tau} = \int_0^{\hat{\theta}^w(n_t)} \frac{\partial x_t^{*,i}}{\partial \tau} f(\theta_i) d\theta_i + x_t^{*,i}|_{\theta_i=\hat{\theta}^w(n_t)} \frac{\partial \hat{\theta}^w(n_t)}{\partial \tau} > 0 \tag{35}$$

$$\frac{\partial \bar{e}_t}{\partial \tau} = \int_0^{\hat{\theta}(n_t,r_t)} \frac{\partial e_t^{*,i}}{\partial \tau} f(\theta_i) d\theta_i + e_t^{*,i}|_{\theta_i=\hat{\theta}(n_t,r_t)} \frac{\partial \hat{\theta}(n_t,r_t)}{\partial \tau} \tag{36}$$

As commented above the second summand vanishes. The first one represents the response of existing evaders. Proposition 2 demonstrates that all individuals will conceal more labor income when taxes increase if the discount rate is sufficiently large. In the case of capital income tax evasion, we can decompose the first summand into a positive effect for the more honest taxpayers ($\tilde{\theta}(n_t, r_t) < \theta_i$) and a negative effect for the less honest ($\tilde{\theta}(n_t, r_t) > \theta_i$) as proposition 1 shows. Thus, the net effect critically depends on how θ_i is distributed, i.e., $F(\theta_i)$. The next proposition studies the effect of an increase in the tax rate on the share of capital income tax evasion when $F(\theta_i)$ is uniformly distributed:

Proposition 6 *Assume that θ_i is uniformly distributed on the interval $[0, \bar{\theta}]$. Then, a higher tax rate increases the share of of capital income tax evasion, $\partial \bar{e}_t / \partial \tau > 0$.*
Proof: *See Appendix.*

Consider that the economy remains in a steady state and that the tax rate increases. A higher tax rate makes tax evasion more profitable which encourages individuals to evade taxes. In our model this implies that the thresholds which determine the number of labor (differentiation of (17)) and capital income tax evaders (according to eq. (22)) increase. Thus, the number of new evaders of both types of income increases. Taxpayers that were evading taxes previous to the rise in the tax rate also react: first, proposition 2 shows that for a relatively large discount factor, all young individuals will conceal more labor income. Second, regarding capital income, proposition 1 shows that evaders with very low moral concerns will evade less whereas evaders with a higher degree of honesty will increase their evasion. It is proved that if θ_i is uniformly distributed, then both the rise of the fraction of (new) capital income evaders and the response of the more honest evaders offset the response of the most dishonest evaders, producing a reduction in the portion of capital income tax evasion. Hence, both labor and capital income tax evasion increase. The increase of the number of both types of evaders will rise the moral cost for the next period which, in turn, will imply a rise in the portion of the evaders and the amount of evades taxes. This process continues so that the dynamic of the economy is characterized by increases in the portion of evaders and tax evasion levels and decreases in tax morale. At the end, the economy reaches a new steady state defined by a larger level of tax evasion and a higher share of evaders.

Empirical evidence supports our findings. Many papers show that tax rates are positively associated with tax evasion in developing and developed countries (Martinez-Vazquez and Rider (2005), Malkawi and Haloush (2008), Dlamini (2017) and Ottone et al. (2018)). Also, Christopoulos (2001) examines Greek data from 1960-1997 and finds that increases in tax rates led to a larger size of the underground economy. More recently, Mitra (2017) shows that once institutional and enforcement variables are controlled for, a strong positive effect of taxes on informality emerges.

3 Productive public expenditures

In our model tax collection is devoted to provide a public good which rises individuals' utility. This is the standard assumption in the literature of tax evasion and economic growth (see, e.g., Levaggi and Menoncin (2013, 2012), Dzhumashev and Gahramanov (2011), Lin and Yang (2001)). However, two studies, Varvarigos (2017) and Chen (2003), consider the possibility of devoting public revenues to provide a productive public expenditure. In these two papers tax evasion produces a clear negative effect on economic growth since it reduces tax collection and so, the provision of the public expenditure which, in turn, jeopardizes economic growth. However, whereas Varvarigos (2017) considers only labor income tax evasion and does not model explicitly the decision of the share of evaded taxes, Chen (2003) focuses on the long term relation between tax evasion and economic growth, disregarding the analysis along the transition towards the steady state.

We now analyze the effect of a productive public expenditure on the relationship between economic growth and tax evasion in our model with both labor and capital income tax evasion. To do so, we consider a production function *a la* Barro, where public expenditure have a positive impact on the productivity of workers, that is,

$$Y_t = A(g_t L_t)^{1-\alpha} K_t^{\alpha}, \tag{37}$$

where g_t is defined by eq. (23).

The rest of the model remains unchanged. In the case of the firms' problem we only need to change the solution to account for the alternative production function. Now, the firms' optimality condition are given by:

$$w_t = (1-\alpha) A k_t^{\alpha} g_t^{1-\alpha} \tag{38}$$

$$r_t = \alpha A k_t^{\alpha-1} g_t^{1-\alpha} \tag{39}$$

If we substitute both the definition of public spending (eq. 23) and the capital market equilibrium condition ($k_t = \bar{s}_{t-1}$) into eq. (39), then the rate of return in equilibrium is given by

$$r_t = \alpha A^{\frac{1}{\alpha}} \left(\tau \left(\alpha [1-(1-p(1+\gamma))\bar{e}_t] + (1-\alpha)[1-(1-q(1+\gamma))\bar{x}_t] \right) \right)^{\frac{1-\alpha}{\alpha}} \tag{40}$$

with $\bar{x}_t = x(n_t)$ defined by eq. (26) and $\bar{e}_t = e(n_{t-1}, r_t)$ defined by eq. (27).

Note that the resulting rate of return is not constant, since the stock of capital depends on the level of tax evasion and on the share of the evaders of the economy in each period. This implies that our model does not behave like a traditional standard model *a la* Barro, in which the economy remains always on a balanced growth path. In other words, the existence of tax evasion implies that the economy evolves over time according to the share of evaders and the amount of evaded taxes.

Therefore, the economy is now characterized by the dynamics of the share of evaders and the rate of return, i.e.,

$$n_{t+1} = \frac{F\left(\hat{\theta}^{w}(n_t)\right) + F\left(\hat{\theta}(n_t, r_t)\right)}{2} \tag{41}$$

$$r_t = \alpha A^{\frac{1}{\alpha}} \left(\tau \left(\alpha[1-(1-(1+\gamma)p)e(r_t, n_t)] + (1-\alpha)[1-(1-q(1+\gamma))x(n_t)]\right)\right)^{\frac{1-\alpha}{\alpha}} \tag{42}$$

The starting levels of evasion in the economy are key to determine the long-run equilibrium since they affect the level of tax collection and so the productivity in the economy. From previous sections we know that insofar countries are accumulating capital, tax morale increases and both the share of evaders and tax evasion decrease. In this model, both the rise in per capita income and the reduction of evasion imply an increase in the effective tax base. This, in turn, leads to an increase in the level of tax collection and in the amount of provided public expenditures. The increase in the public expenditure raises labor productivity, wages and savings, and so, expands capital accumulation. Hence, if the initial amount of evasion is low enough and the (productive) public expenditure are considerably high, the economy would reach a long-run equilibrium with a small number of evaders, a low level of tax evasion and a high level of per capita income. On the contrary, if the starting levels of the share of evaders and tax evasion are big enough, the amount of public spending could be small enough to promote capital accumulation. The low level of public expenditure would produce a low productivity in the economy which, it turn, would brake economic growth. This implies that the economy might be trapped in an equilibrium characterized by a large portion of evaders, a high level of tax evasion and a low amount of per capita capital. Varvarigos (2017) and Chen (2003) obtain a similar result. Moreover Chen (2003) shows that, in order to obtain sustained levels of income, the public good has to be efficiently enough to offset low levels of tax collection.

4 Other Features

In this section we study several robustness checks in order to show that our findings are robust to assumptions and modeling choices made. More precisely, we discuss how the consideration of a direct pecuniary cost of evasion (section 5.1) and a cost of audit and enforcement (section 5.1) would affect the main results of our model.

4.1 Pecuniary cost of tax evasion

As most of papers on tax morale, our model does not consider direct pecuniary costs of tax evasion. Since our interest is in analyzing the effects of tax morale on tax evasion, we have focused on non-pecuniary moral costs of evasion. Now, we analyze if our results are robust to the consideration of these pecuniary costs of evasion. Following Chen (2003), imagine that individuals face transaction costs when evading taxes. The transaction costs may include bribing tax officials, hiring lawyers to elude taxes, etc. It is reasonable to assume that these transaction costs increase with the share of concealed income (see Chen (2003)). Thus, they can formalized as ξx_t and ηe_{t+1} for labor and capital income tax evasion, respectively, with $\xi, \eta > 0$ as cost parameters[17]. Consumption levels in the first period for both states, getting detected (d-state) and remaining undetected (u-state), are now

$$c_t^d = w_t(1 - \tau - \gamma\tau x_t - \xi x_t) - s_t$$

$$c_t^u = w_t(1 - \tau + \tau x_t - \xi x_t) - s_t$$

whereas that equivalent ones for the second period are

$$d_{t+1}^u = (R_{t+1}^u - r_{t+1}\eta e_{t+1})s_t$$

$$d_{t+1}^d = (R_{t+1} - r_{t+1}\eta e_{t+1})^d s_t$$

Note that the existence of interior solutions for both labor and capital income tax evasion requires in both cases gambles better than fair. In the baseline model this implies $1 - q(1+\gamma) > 0$ and $1 - p(1+\gamma) > 0$, respectively. Now, including the pecuniary cost of evasion, these conditions result to be $1 - q(1+\gamma) > \eta/\tau$ and $1 - p(1+\gamma) > \eta/\tau$. Thus, interior solutions require the cost parameters, ξ and η, or alternatively, the probabilities of detecting evasion, p, q, and/or the fine, γ, to be sufficiently small. As the remainder of the analysis is qualitatively very similar, we conclude that our results are robust to considering pecuniary costs of tax evasion.

4.2 Cost of audit

As the majority of papers in the literature on tax morale, we also abstract from enforcement/audit costs. Specifically, in our model tax collection is entirely devoted to finance

[17] Note that ξ and η might have different values, since labor income tax evasion might involve different skills and practices than capital income tax evasion.

a public good. In this section we study the implications of considering that the audit activity is costly and so, that the government has to devote public resources in order to finance it. Consider the baseline version of the model where the public good produces utility. Imagine that a share, $\lambda > 0$, of the tax collection is used to finance the audit technology for tax evasion in capital income and that the cost of audit is defined by $\nu h(p_t)$, with $\nu > 0$ as a cost parameter and $h' > 0$. Then, $\lambda T_t = \nu h(p_t)$. We might rewrite the probability of detecting capital income tax evasion at period t as,

$$p_t = \phi\left(\frac{\lambda}{\nu}T_t\right),$$

with $\phi' > 0, \phi'' < 0$ and $\phi \in (0,1)$. Similarly, for labor income tax evasion, consider that the government devotes a share $\epsilon > 0$ of the tax collection to finance the cost of audit it, denoted by $\sigma b(q_t)$, with $\sigma > 0$ as a cost parameter and $b' > 0$. We then obtain

$$q_t = \psi\left(\frac{\epsilon}{\sigma}T_t\right),$$

with $\psi' > 0, \psi'' < 0$ and $\psi \in (0,1)$. Note that in this setting the capital accumulation process positively affects the level of the tax collection and so, the probability of detection. Let us now study the dynamics of the economy: insofar countries are accumulating capital, and the interest rate decreases, the share of capital income evaders also declines. The reduction of capital income evaders increases the moral cost which, in turn, discourages capital and also labor income tax evasion even more. Thus, tax evasion declines whereas the economy is growing. Consequently, both increases in income (i.e., the tax base) and reductions of tax evasion along the transition yield an increase in the tax revenues and so the audit probability. This rise in the probability of detection would reinforce the decrease in the share of evaders and so the effects described above. Therefore, the qualitative results of the paper would remain unchanged.

In a more sophisticated setting, where the public spending affects the productivity of the economy or the government decides the optimal tax rate and how to distribute the tax collection among alternative uses (see, for instance, Chen (2003) or Bethencourt and Kunze (2015)), results, however, might change. Consider, e.g., the version of the model we study in section 4, where tax collection is used to provide a productive public good. In this section we showed that if both the starting number of evaders and the level of tax evasion are large enough, then the amount of public expenditure is substantially diminished. This, in turn, might substantially reduce the productivity in the economy thereby harming economic growth. The economy would be stuck in an equilibrium with a high tax evasion level, a high portion of evaders and a low level of per capita capital (Varvarigos (2017) obtains a similar result). Imagine now that the government also finances tax enforcement. In this case, a bad technology of detection might reinforce the previous result. That is, even when the public good is highly productive, if the productivity of enforcement is low and the share of the tax collection devoted to tax enforcement is high, then the economy would be trapped in a low steady state. As in Chen (2003), the public good has to be efficiently enough to offset low levels of enforcement productivity and to encourage capital accumulation.

5 Conclusions

This paper incorporates a social norm towards tax compliance into an overlapping generations model with both labor and capital income tax evasion. It is shown that the existence of a social norm generates a complementarity of both types of evasion in the economy: A lower share of evaders for one of these types strengths the social norm which, in turn, reduces incentives to evade both capital and labor income taxes. This complementarity reinforces the decline of tax evasion and the share of evaders when countries accumulate capital, thereby generating a novel channel through which (capital) income tax evasion may affect economic growth.

Our model helps to explain many issues surrounding the relationship between economic development and tax evasion by generating several predictions which are supported by empirical evidence. First, a positive relation between capital and labor income tax evasion, i.e., countries with high levels of labor income tax evasion also display high levels of capital income tax evasion; second, a negative relationship between tax evasion and per capita income, i.e., richer countries have lower levels of capital and labor income tax evasion whereas poorer countries have high levels of both types of tax evasion and; third, decreasing shares of capital and labor income tax evasion when countries grow and tax morale increases. In addition, we show that a higher tax rate increases the share of capital and labor income tax evaders as well as aggregate evasion. These results are key in designing programs aimed at fostering compliance and discouraging tax evasion. They imply that besides economic effects, polices aimed to deter evasion also generate amplifying and lasting effects through their impact on the evolution of social norms along the development process and in the long run.

The paper points out the importance of social norms in shaping the evolution of the share of evaders in the society and the level of tax evasion. However, alternative explanations of these dynamics, such as the quality of institutions, can be found in the literature. In this regard, Torgler and Schneider (2009), for instance, show that there is a negative correlation between the quality of institutions and the size of the shadow economy, as well as between tax morale and the size of the shadow economy. Advanced economies show on average high quality institutions which make them more efficient collecting taxes (for instance, due to better informed tax officers (Kleven et al. (2011))). As these countries devote a fraction of public revenues to quality improvements (see Bethencourt and Kunze (2015)), this allows us to conclude that the quality of institutions and tax morale reinforce each other.

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Appendix

Proof of proposition 1:

We obtain the explicit solution of $e_t^{i,*}$ from eq. (12) as[18]

$$e_t^{i,*} = \frac{\beta r_t \gamma \tau - R_t(\gamma-1)(\theta_i + \mu(1-n_t)) - \tilde{m}}{2 r_t \gamma \tau (\theta_i + \mu(1-n_t))} \tag{A.1}$$

with $\tilde{m} = \sqrt{(\beta\tau\gamma r_t - (\theta_i + (\gamma+1)R_t\mu(1-n_t)))^2 + 4p\beta\tau\gamma r_t(\theta_i + (\gamma+1)R_t\mu(1-n_t))}$ and $R_t = 1 - \delta + (1-\tau)r_t$.

We then derive $e_t^{i,*}$ with respect to τ and r_t, that is,

$$\frac{\partial e_t^{i,*}}{\partial \tau} = \frac{\tilde{m}(1-\delta+r_t)(\gamma-1) - (\gamma+1)((\theta_i + \mu(1-n_t))R_t(\gamma+1) + \beta r_t \gamma \tau (1-2p))}{2 r_t \gamma \tau^2 \tilde{m}} \tag{A.2}$$

[18]The solution of this equation produces two values of $e_t^{i,*}$. However, one of them turns out to be positive for all values of θ_i, which is economically inconsistent with the model, Hence, we disregard such a solution.

$$\frac{\partial e_t^{i,*}}{\partial r_t} = \frac{\tilde{m}(1-\delta)(\gamma-1)+(\gamma+1)((\theta_i+\mu(1-n_t))R_t(\gamma+1)-\beta r_t\gamma\tau(1-2p))}{2r_t^2\gamma\tau\tilde{m}} \tag{A.3}$$

From these expressions, we obtain

$$\frac{\partial e_t^{*,i}}{\partial r_t}\begin{cases} \leq 0 & \text{if } \theta_i \leq \tilde{\theta} \\ > 0 & \text{if } \tilde{\theta} < \theta_i \end{cases} \tag{A.4}$$

$$\frac{\partial e_t^{*,i}}{\partial \tau}\begin{cases} \leq 0 & \text{if } \theta_i \leq \tilde{\theta} \\ > 0 & \text{if } \tilde{\theta} < \theta_i \end{cases} \tag{A.5}$$

where

$$\tilde{\theta} = -\mu(1-n_t) + \frac{\beta\tau r_t\sqrt{\gamma}(\sqrt{p(1-p)}(\gamma-1)-\sqrt{\gamma}(1-2p))}{(1+\gamma)R_t} \tag{A.6}$$

Finally, from eq. (20) we get

$$\hat{\theta} > \tilde{\theta} \quad \Leftrightarrow \quad < \gamma\frac{1-p}{p}. \tag{A.7}$$

In order to ensure the existence of an interior solution, we assume that this conditions holds throughout the whole paper.

Proof of proposition 2:

In a first step, we need to obtain the formal expression of $x_t^{i,*}$. To do so, we have to solve the system of two equations defined by (10) and (12). However, given that we are not able to obtain a closed form solution, we proceed by calculating implicitly those derivatives. Specifically, we define the following system of implicit equations:

$$F = \left[(1-q)u'(c_t^u)+qu'(c_t^d)\right]-\frac{\beta}{s_t} \tag{A.8}$$

$$G = \tau w_t\left[(1-q)u'(c_t^u)-\gamma qu'(c_t^d)\right]-(\theta_i+\mu(1-n_t)) \tag{A.9}$$

We then obtain the Jacobian matrix of the system as

$$\frac{\partial(F,G)}{\partial(x_t,s_t)} = \begin{pmatrix} F'_s & F'_x \\ G'_s & G'_x \end{pmatrix} \tag{A.10}$$

$$F'_s = \left[\frac{1-q}{(c_t^u)^2} + \frac{q}{(c_t^d)^2}\right] + \frac{\beta}{s_t^2} > 0 \tag{A.11}$$

$$F'_x = -\left[\frac{1-q}{(c_t^u)^2} - \frac{q\gamma}{(c_t^d)^2}\right](\tau w_t) \tag{A.12}$$

$$G'_s = -F'_x \tag{A.13}$$

$$G'_x = -\left[\frac{1-q}{(c_t^u)^2} + \frac{q\gamma^2}{(c_t^d)^2}\right](\tau w_t)^2 < 0 \tag{A.14}$$

It is straight forward to see that the sign of the determinant of the Jacobian matrix is negative, that is, $|J| = F'_s G'_x - F'_x G'_s < 0$, since $|F'_s G'_x| > |F'_x G'_s|$. We then apply the Cramer rule for obtaining the derivatives:

$$\frac{\partial x_t^{i,*}}{\partial w_t} = \frac{\begin{vmatrix} F'_s & -F'_{w_t} \\ G'_s & -G'_{w_t} \end{vmatrix}}{|J|} = \frac{F'_{w_t} G'_s - F'_s G'_{w_t}}{|J|} \tag{A.15}$$

$$F'_{w_t} = -\left[\frac{(1-q)(1-\tau+\tau x_t)}{(c_t^u)^2} + \frac{q(1-\tau-\gamma\tau x_t)}{(c_t^d)^2}\right] < 0 \tag{A.16}$$

$$G'_{w_t} = \tau\left[\frac{(1-q)}{c_t^u} - \frac{q\gamma}{c_t^d} - w\left(\frac{(1-q)(1-\tau+\tau x_t)}{(c_t^u)^2} - \frac{q\gamma(1-\tau-\gamma\tau x_t)}{(c_t^d)^2}\right)\right] \tag{A.17}$$

Since $F'_{w_t}G'_s - F'_sG'_{w_t} = 0$, we obtain $\frac{\partial x_t^{*,i}}{\partial w_t} = 0$. Similarly,

$$\frac{\partial x_t^{i,*}}{\partial \tau} = \frac{\begin{vmatrix} F'_s & -F'_\tau \\ G'_s & -G'_\tau \end{vmatrix}}{|J|} = \frac{F'_\tau G'_s - F'_s G'_\tau}{|J|} \tag{A.18}$$

$$F'_\tau = \left[\frac{(1-q)(1-x_t)}{(c_t^u)^2} + \frac{q(1+\gamma x_t)}{(c_t^d)^2}\right] w_t > 0 \tag{A.19}$$

$$G'_\tau = w_t\left[\frac{(1-q)}{c_t^u} - \frac{q\gamma}{c_t^d} + \tau w_t\left(\frac{(1-q)(1-x_t)}{(c_t^u)^2} - \frac{q\gamma(1+\gamma x_t)}{(c_t^d)^2}\right)\right] \tag{A.20}$$

Note that we can rewrite G'_τ as

$$G'_\tau = w_t(w_t - s_t)G'_s \tag{A.21}$$

and so,

$$\frac{\partial x_t^{i,*}}{\partial \tau} = \frac{F'_\tau G'_s - w_t(w_t - s_t)F'_s G'_s}{|J|} \tag{A.22}$$

Since $|F'_\tau G'_s| \leq |F'_s G'_\tau|$, the sign of the numerator of expression (A.22) depends on the sign of G'_τ, that is, the sign of G'_s. Note that

$$G'_s > 0 \Leftrightarrow \underline{\beta} = \frac{q(1+\gamma)(w_t(1-\tau) - s_t)s_t}{(c_t^d)^2} < \beta \tag{A.23}$$

Therefore, $\frac{\partial x_t^{*,i}}{\partial \tau} > 0$ when $\underline{\beta} < \beta$.

Proof of proposition 3:

We depart from the Jacobian defined in the proof of proposition 2. We then apply the Cramer rule to obtain the derivatives:

$$\frac{\partial s_t^{i,*}}{\partial n_t} = \frac{\begin{vmatrix} -F'_{n_t} & F'_x \\ -G'_{n_t} & G'_x \end{vmatrix}}{|J|} = \frac{F'_x G'_{n_t} - F'_{n_t} G'_x}{|J|} = \frac{F'_x G'_{n_t}}{|J|} \tag{A.24}$$

$$F'_{n_t} = 0 \tag{A.25}$$

$$G'_{n_t} = \mu > 0 \tag{A.26}$$

Substituting these derivatives yields

$$\frac{\partial s_t^{i,*}}{\partial n_t} = \frac{F'_x G'_{n_t}}{|J|} = -\frac{G'_s G'_{n_t}}{|J|}. \tag{A.27}$$

Since $G'_s > 0$ when $\underline{\beta} < \beta$, then $\frac{\partial s_t^{i,*}}{\partial n_t} > 0$.

Similarly,

$$\frac{\partial s_t^{i,*}}{\partial k_t} = \frac{\begin{vmatrix} -F'_{k_t} & F'_x \\ -G'_{k_t} & G'_x \end{vmatrix}}{|J|} = \frac{F'_x G'_{k_t} - F'_{k_t} G'_x}{|J|} \tag{A.28}$$

$$F'_{k_t} = F'_{w_t} \frac{\partial w_t}{\partial k_t} < 0 \tag{A.29}$$

$$G'_{k_t} = G'_{w_t} \frac{\partial w_t}{\partial k_t} \tag{A.30}$$

Thus, we can rewrite the above expression as

$$\frac{\partial s_t^{i,*}}{\partial k_t} = \frac{F_k'\left(\frac{F_x' G_{k_t}'}{F_k'} - G_x'\right)}{|J|} = \frac{F_k'\left(\frac{F_x' G_{w_t}'}{F_w'} - G_x'\right)}{|J|} \tag{A.31}$$

Using $F_{w_t}' G_s' - F_s' G_{w_t}' = 0$ from the proof of proposition 2, then

$$\frac{\partial s_t^{i,*}}{\partial k_t} = \frac{\frac{F_k'}{F_s'}\left(F_x' G_s' - G_x' F_s'\right)}{|J|} = -\frac{F_k'}{F_s'} > 0 \tag{A.32}$$

Proof of proposition 4:

Assume that condition (30) holds. Define $n_{t+1} = g(n_t) = \frac{F\left(\hat{\theta}^w(n_t)\right) + F\left(\hat{\theta}(n_t, r_*)\right)}{2}$ and $h(n_t) = g(n_t) - n_t$ from eq. (25). We then have $h(0) = \frac{F\left(\hat{\theta}^w(1) - mu\right) + F\left(\hat{\theta}(k_*, 1) - \mu\right)}{2} > 0$, since $\mu < \hat{\theta}^w(1)$, $\mu < \hat{\theta}(k_*, 1)$ and $h(1) = \frac{F\left(\hat{\theta}^w(1)\right) + F\left(\hat{\theta}(k_*, 1)\right)}{2} - 1 < 0$ since $\hat{\theta}^w(1) < \bar{\theta}$ and $\hat{\theta}(k_*, 1) < \bar{\theta}$. The Bolzano theorem implies that there must be a point $n_* \in (0,1)$ such that $h(n_*) = 0$. From the definition of $h(.)$ this implies $n_{t+1} = n_t = n_*$, so that n_* is an equilibrium.

Proof of equation (31):

The dynamics of the economy are described by the following system of two equations:

$$n_{t+1} = \Psi(n_t, k_t) \tag{A.33}$$

$$k_{t+1} = \Phi(n_t, k_t). \tag{A.34}$$

Following de la Croix and Michel (2002), the steady state of the system above, denoted by (k_*, n_*), is a saddle if the following condition is verified:

$$|T| > |1 + D|$$

where $T = \Phi_{n_t}(n_*, k_*) + \Psi_{k_t}(n_*, k_*)$ is the trace of the Jacobian matrix G obtained from the first-order Taylor expansion of the dynamic system evaluated at the steady state, i.e.

$$G = \begin{pmatrix} \Phi_{n_t}(n_*, k_*) & \Phi_{k_t}(n_*, k_*) \\ \Psi_{n_t}(n_*, k_*) & \Psi_{k_t}(n_*, k_*) \end{pmatrix} \tag{A.35}$$

and $D = \Phi_{n_t}(n_*, k_*)\Psi_{k_t}(n_*, k_*) - \Phi_{k_t}(n_*, k_*)\Psi_{n_t}(n_*, k_*)$, with

$$\Phi_{n_t}(n_*, k_*) = \int_0^{\hat{\theta}^w(n_t)} \frac{\partial s_t^{*,i}}{\partial n_t} f(\theta_i) d\theta_i + s_t^{*,i}\Big|_{\theta_i = \hat{\theta}^w(n_t)} \frac{\partial \hat{\theta}^w(n_t)}{\partial n_t} > 0 \tag{A.36}$$

$$\Psi_{k_t}(n_*, k_*) = \frac{1}{2} F'(\hat{\theta}(n_*, k_*)) \frac{\partial \hat{\theta}}{\partial r_t} \frac{\partial r_t}{\partial k_t} < 0 \tag{A.37}$$

$$\Phi_{k_t}(n_*, k_*) = \int_0^{\bar{\theta}} \frac{\partial s_t^{*,i}}{\partial n_t} f(\theta_i) d\theta_i > 0 \tag{A.38}$$

$$\Psi_{n_t}(n_*, k_*) = \frac{1}{2} F'(\hat{\theta}^w(n_*, k_*)) \frac{\partial \hat{\theta}^w}{\partial n_t} + \frac{1}{2} F'(\hat{\theta}(n_*, k_*)) \frac{\partial \hat{\theta}}{\partial n_t} > 0 \tag{A.39}$$

since $\frac{\partial s_t^{*,i}}{\partial n_t} > 0$ and $\frac{\partial s_t^{*,i}}{\partial k_t} > 0$ by proposition 2, $\frac{\partial \hat{\theta}^w(n_t)}{\partial n_t} = \mu > 0$, $\frac{\partial r_t}{\partial k_t} = -\alpha(1-\alpha)Ak_*^{\alpha-2} < 0$, $\frac{\partial \hat{\theta}}{\partial r_t} = \frac{\tau(1-\delta)(1-p(1+\gamma))}{(1-\delta+(1-\tau)r_*)^2} > 0$ and $\frac{\partial \hat{\theta}^w}{\partial n_t} = \frac{\partial \hat{\theta}}{\partial n_t} = \mu > 0$. The condition $|1 + D| > |T|$ is equivalent to

$$\frac{\mu \Phi_{k_t}(n_*, k_*)}{2(1 - \Phi_{n_t})} \Big(F'(\hat{\theta}^w(n_*, k_*)) + F'(\hat{\theta}(n_*, k_*)) \Big) + \frac{\frac{\partial \hat{\theta}}{\partial r_{t+1}} \alpha(1-\alpha)Ak_*^{\alpha-2}}{2} F'(\hat{\theta}(n_*, k_*)) > 1. \tag{A.40}$$

Therefore, eq. (31) is proved.

Proof of propositions 5 and 6:

In proposition 1 we obtained the derivatives of $e_t^{i,*}$ with respect to r_t and τ (see eqs. A.3 and A.2 respectively). Since we are interested in the aggregate effects, we integrate these derivatives over the relevant interval $[0, \hat{\theta}]$, where we have assumed that θ_i is uniformly distributed. We then obtain:

$$\frac{\partial \bar{e}_t}{\partial r_t} = \frac{\left[\beta r_t \gamma \tau - \tilde{m} - \mu(1-n_t)R_t(\gamma-1)\right](1-\delta)}{2R_t r_t^2 \gamma \tau} \tag{A.41}$$

$$\frac{\partial \bar{e}_t}{\partial \tau} = \frac{\left[\beta r_t \gamma \tau - \tilde{m} - \mu(1-n_t)R_t(\gamma-1)\right](1-\delta+r_t)}{2R_t r_t \gamma \tau^2} \tag{A.42}$$

Note that both equations have the same expressions in square brackets. From eq. 32 and $dr_t/dk_t < 0$, we know that the sign of the square brackets is always positive. Since $(1-\delta+r_t) > 0$ we get

$$sign\left(\frac{\partial \bar{e}_t}{\partial k_t}\right) = -sign\left(\frac{\partial \bar{e}_t}{\partial \tau}\right) \tag{A.43}$$

Hence, $\frac{\partial \bar{e}_t}{\partial \tau} > 0$ (which proves proposition 5) and $\frac{\partial \bar{e}_t}{\partial k_t} < 0$ (which proves proposition 6).