The Role of Institutions in Shaping the Growth-Aid Relationship*

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Abstract

Empirical evidence on the relationship between aid and economic growth is mixed and inconclusive. This paper proposes a theory to explain these contradictory findings. We build a growth model with a productive public good and agents that devote time to appropriate public resources. Aid increases public resources, raising the provision of the productive public good, but promotes rent-seeking. As recent empirical evidence suggests, a hump-shaped relationship between aid and growth emerges: too much aid is counterproductive for growth, particularly when institutions are weak. Aid transmits growth from the donor to the receptor country but harms income convergence and even prevents convergence among ex-ante identical countries when aid exceeds a certain threshold. Institutional improvements raise such a threshold. Thus, countries with lower income and lower institutional quality should receive less aid, unless an institutional reform is taken as a previous step to receive that aid.

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1. Introduction

"Does international aid have positive effects on growth?" The empirical literature on growth and aid has traditionally not offered any conclusive answer. Whereas some papers document positive effects, others find negative or null impacts[1]. This mixed and ambiguous evidence has motivated the emergence of new lines of research. Regarding the empirical literature, a new set of papers explore the existence of non-linearities in the aid-growth relationship. Unlike the first contributions, which documented negative or positive relationships between these variables, these papers find evidence of diminishing returns to aid in promoting growth and, related to this, strong evidence of a hump-shaped relationship between aid and growth[2]. Also, another group of papers reveals that foreign aid stimulates the emergence of rent-seeking activities by powerful social groups to appropriate government resources[3]. Finally, another branch of the literature documents that aid may erode the quality of institutions[4], which would harm growth[5].

Regarding the theoretical literature, in contrast to the empirical one, the number of papers analyzing the link between growth, governance, and aid has been scarce[6]. Indeed, as Temple (2010) states, most studies have typically omitted considerations such as governance and political economy, which play a crucial role in the debate about aid effectiveness. This paper aims to fill this gap in the literature by presenting a growth model that sheds some light on the empirical facts mentioned above, helping to understand the diversity in the research findings on the impacts of foreign aid on economic growth. Thus, we focus on the effect of international aid on growth and the convergence among countries, providing guidance for the design of aid policies.

We propose a model in which agents devote time to work and rent-seeking activities to appropriate public resources from the government. There are two sources of public resources: tax collection obtained from non-distortionary taxes and foreign aid. The public revenue not grabbed by rent-seeking activities is devoted to financing a public good, which is productive *à la* Barro and generates endogenous growth.

We show that despite non-distortionary taxes, a hump-shaped relationship exists between aid and growth. This result is due to three offsetting mechanisms: (*i*) an increase in aid raises the government's resources and so the provision of public goods,

[1] See detailed revisions of the literature by Minoiu and Reddy (2010), Clemens et al. (2011), and Hatlebakk (2021).

[2] See Hansen and Tarp (2001) and Clemens et al. (2011).

[3] See Maren (1997) and Reinnika and Svensson (2004). More recently, Ravallion (2014) argues that though external aid is targeted at poor people, it goes through a government that does not share that goal. This feature explains why the purpose of aid has been thwarted at times.

[4] See Rajan and Subramanian (2007), Djankov et al. (2008), Kangoye (2013), and Asongu (2015).

[5] See Djankov et al. (2008) for an excellent literature discussion.

[6] Section 2 of the paper reviews the literature on the relationship between aid and growth, both theoretical and empirical literature.

expanding the productivity of the private sector and thus, increasing the growth rate; (*ii*) the increase in the government's resources improves the profitability of rent-seeking activities, which reduces resources devoted to the public good and so the growth rate and; (*iii*) because agents devote more time to rent-seeking activities, labor assigned to production drops, harming growth. We show that the first effect prevails for low aid levels, while the other two are stronger for high aid levels. Hence, international aid is beneficial for the growth of the receptor country when aid is not too large. Furthermore, we show that there is an "optimal" level of international aid, which maximizes the growth rate of the receptor country. If international aid is above this threshold, aid becomes detrimental to growth.

Institutions play a crucial role: an improvement in the quality of institutions affects growth positively in two different ways. First, it reduces the reward for rent-seeking activities, increasing the time devoted to production and the growth rate. Second, it reduces the effect of international aid on promoting rent-seeking, raising the optimal level of international aid that maximizes the growth rate. Consequently, countries with bad institutions not only grow slower but are also the ones with a stronger "rent-seeking promoting effect" of aid, in which it is more likely that aid turns out to be counterproductive for growth.

We modify the model to analyze the effect of international aid on convergence across countries. To do this, we introduce two countries, the North and the South, and we consider two alternative settings: First, a two-different countries model, in which countries have different parameter values (productivity, institutional quality, etc.) and; second, a two-alike countries model, in which countries are identical in everything except for their initial capital.

In the two-different countries setting, the North is the country that would grow faster in the absence of international aid, and it is the donor of international aid. International aid equalizes the growth rate of the South and the North but not the income levels. There are two balanced growth paths: the"good" balanced growth path (good BGP) and the "bad" one (bad BGP). The convergence rate, defined as the ratio between income in the South and the North, is higher on the good BGP than on the bad BGP. Excessive international aid may undermine the convergence of the South to the North since international aid promotes rent-seeking. Indeed, reducing the level of international aid below a certain threshold avoids the convergence of the South to the bad BGP. Moreover, such a threshold shrinks with the weakness of institutions. Thus, aid is more likely to have a counterproductive effect in countries with weaker institutions. Accordingly, if aid policy aims to make countries converge to the good BGP, aid should be less generous in countries with weaker institutions and, consequently, a paradox arises: those countries with weaker institutions and lower per capita income should receive less international aid to avoid harming their growth and convergence with the rent-seeking promoting effect from aid. Therefore, if aid aims to foster convergence, the receptor country should implement institutional reform as a previous step to receiving aid.

In the the two-alike countries model, countries are identical in everything except for their initial capital and the richer country donates international aid to the poorer one. International aid may prevent the convergence of both countries to the same BGP or "symmetric" BGP, especially when institutions are weak. The reason is that international aid promotes rent-seeking in the receptor country, generating an "asymmetric" BGP where there is no convergence in per capita income and, consequently, international aid from the donor to the receptor country self-perpetuates. Along this asymmetrical BGP, rent-seeking in the receptor country is higher than in the symmetric one due to international aid. Furthermore, the donor country diverts part of its public resources from productive public investment to international aid. Consequently, the growth rate on the asymmetric BGP is lower than that of the symmetric BGP, in which countries converge in per capita income and, as a result, there is no need for international aid. Therefore, aid harms both growth and convergence in the asymmetric BGP.

This paper can be framed in the recent literature that links aid with growth, considering the role of aid-financed public investment or infrastructure (see, among others, Chatterjee et al., 2003, Chatterjee and Turnovsky, 2004, 2005 and Agénor and Yilmaz, 2013). However, in contrast with existing papers, we analyze the incentive problems that aid may produce in reallocating resources to rent-seeking. The rent-seeking mechanism and the productive role of the aid generate the hump-shaped relationship between growth and aid in our model.

This paper is also related to the literature on the impact of aid on rent-seeking activities. In this regard, Svensson (2000) presents a repeated game model in which different groups interact strategically to capture the aid (income) received by the government. Dalgaard and Olsson (2008) consider a similar setting in which the elite and the rest of the citizenry compete for a given share of the appropriate- able aid (income) and where aid is assumed affecting positively to productivity. They find a hump-shaped relationship between total income and aid only when the receptor country's institutional strength is low enough. Nevertheless, no one of these studies analyzes the consequences of rent-seeking on growth, which is the goal of the current paper. The only article that considers this issue is Hodler (2006). He introduces aid and rent-seeking into a la Barro (JPE 1990) growth model and finds a well-defined positive relationship between economic growth and aid. Hence, that model cannot capture the non-linearities and the hump-shaped relationship that has been documented recently. Also, since aid is a linear function of the income of the receptor country, both the time devoted to rent-seeking and the growth rate result to be always constant. Thus, there is no transition dynamic in the model. Furthermore, the donor country is not included in the analysis, so the convergence of the receptor to the donor country cannot be analyzed. Consequently, it is impossible to provide guidance in the design of aid policies that foster convergence[7]

The rest of this paper is organized as follows. Section 2 revises empirical and theoretical literature on aid and economic growth. Section 3 presents the basic model, where individuals devote time to rent-seeking and work, and the government uses tax collection and international aid to finance a productive good. Section 4 analyzes the behavior and decisions of agents in the long run and characterizes the balanced growth path. Section 5 studies the income convergence of two different countries: the North, the one with a larger growth rate in the absence of convergence and donor of international aid, and the South, the receptor country. Section 6 analyzes the effect of international aid on income convergence in an alternative model with two-alike countries. Finally, Section 7 concludes. All the technical details and proof are included in the Appendix.

2. Review of the literature about aid and economic growth relationship

2.1. Empirical literature

Former empirical studies about aid and economic growth are not conclusive about the effectiveness of aid promoting growth. A first group of papers finds that aid generates a positive effect on growth because it fosters public and private investment (see, among others, Burnside and Dollar, 2000; Dalgaard et al., 2017; and Galiani et al., 2010). Another group of papers documents that aid undermines growth by stimulating rent-seeking activities by powerful social groups (see, for instance, Maren, 1997; Reinnika and Svensson, 2004; and Ravallion, 2014) or eroding institutional quality, for example, increasing corruption (Alesina and Weder, 2002; Svensson, 2000; Rajan and Subramanian, 2007; Djankov et al., 2008; and Asongu, 2015). Finally, a third group of papers reports that aid works under some conditions (Burnside and Dollar, 2000; Collier and Dollar, 2008, among others). Hence, whereas some studies document that the positive effects of aid prevail over adverse effects, others report the opposite.

This ambiguous result about the effect of aid on growth motivates the emergence of a new branch in the literature, which explores non-linearities in the aid-growth relationship (see Hansen and Tarp, 2001; Clemens et al., 2011). In a very influential study, Hansen and Tarp (2001) find diminishing returns to aid in promoting growth. Related to this, they also find strong evidence of a hump-shaped relationship between aid and growth. Following Hansen and Tarp (2001), a new group of papers investigates a possible quadratic relationship between aid and growth, providing consistent evidence of a hump-shaped relationship[8]. An interesting recent contribution is Bandyopadhyay et

[7] There are many other aspects in the literature on external aid which are not so closely linked with our model, like political economy issues, transaction costs, or agency problems. For an excellent survey, see Paul (2006).

[8] See Feeny and McGillivray (2009) for a thorough survey of this literature.

al. (2013), which found that the relationship between aid and growth is hump-shaped, confirming the diminishing returns that Hansen and Tarp (2001) found. However, they find that this relationship is U-shaped when they test the loans-growth relationship along the lines of the finance-growth literature. This outcome suggests that when aid is assigned to the local government directly without any (financial) clause, the risk of rent-seeking activities increases considerably. Thus, abundant aid renders in high levels of rent-seeking that undermines growth opportunities.

Finally, Feeny and McGillivray (2009) go further into the analysis, proposing and estimating a measure of efficient aid. They compare efficient and real aid and find that many countries have been over-aided and that excessive aid has not allowed receptor countries to use it efficiently to promote growth. They estimated negative economic growth rates resulting from the excessive amount of aid. Thus, the opportunity cost of the overmuch aid seems quite significant in terms of the foregone growth gains from the efficient amount of aid.

2.2. Theoretical literature

The first theoretical contributions in the literature that links aid and growth use the neoclassical capital accumulation framework, assuming that aid is distributed among agents as transfers. There are two approaches. The first one considers the Ramsey-Cass-Koopmans model (see 0bstfeld, 1999, Scholl, 2009, and Arellano et al., 2009). In this environment, an increase in aid raises the per capita consumption of the receptor country without affecting its steady state per capita capital. Indeed, an increase of aid in the infinite-horizon household model represents a pure wealth effect that increases the present and future consumption without having any substitutive effect, that is, without affecting the reward for saving. So, aid does not affect per capita capital at the steady state. The second approach considers Diamond's model (see Dalgaard, Hansen, and Tarp, 2004). In this framework, the effect of aid on per capita capital is not well determined, given that transfers in the first period of life encourage saving and capital accumulation, while transfers in the second period of life have just the opposite effect.

More recent literature considers the most plausible case in which aid is not transferred to consumers but used for public expenditure projects intended to increase the economy's productivity. For instance, Chatterjee et al. (2003) and Chatterjee and Turnovsky (2005) show that whereas untied aid transfers have no dynamic impacts, both permanent and transitory aid transfers tied to investment in public infrastructure may lead to welfare improvements. In addition, Chatterjee and Turnovsky (2005) show that when the labor supply is flexible, untied aid transfers might even produce adverse effects if they reduce work effort. Also, Agénor and Yilmaz (2013) find similar results but consider the possibility of having two different types of productive public goods.

Finally, many papers have studied the effect of rent-seeking activities on the economy (see Bethencourt and Perera-Tallo, 2015, for a review). However, the number of studies that have analyzed the impact of rent-seeking on the aid-growth relationship is scarce. An exception is Hodler (2006), who finds a well-defined positive relationship between aid and economic growth. Nevertheless, he cannot explain the hump-shaped relation, which is documented empirically.

Therefore, we can conclude that, unlike our paper, the theoretical literature on the aid and economic growth relationship: first has barely considered the key role of rent-seeking in shaping that relation; second, has found only clear monotone relationships, omitting the existence of hump-shaped relations between these variables (as the most recent empirical evidence documents it) and third; has not dealt with the convergence issue among countries.

3. The Model

Time is continuous with an infinite horizon. Population, $N(t)$, is constant. There is a single good in the economy that can be used for consumption, investment, and as a public good provided by the government:

$$y(t) = c(t) + g(t) + \dot{k}(t) + \delta k(t) \tag{3.1}$$

where y denotes per capita production, c denotes per capita consumption, g denotes per capita public good provided by the government, k denotes per capita capital, and $\delta \in (0,1)$ denotes the depreciation rate.

3.1. Preferences

There is a continuum of identical households indexed in $[0,1]$ with preferences given by a time-separable utility function:

$$\int_0^\infty u(c(t))e^{-\rho t}dt \tag{3.2}$$

where $\rho > 0$ is the discount rate of the utility function, c denotes the consumption, and $u(.)$ is the isoelastic felicity function:

$$u(c) = \begin{cases} \frac{c^{1-\sigma}}{1-\sigma} & if \quad \sigma \in (0,1) \cup (1,+\infty) \\ ln(c) & if \quad \sigma = 1 \end{cases} \tag{3.3}$$

3.2. Production technology

The technology is given by the following production function:

$$y(t) = Ag(t)^{1-\alpha} K(t)^{\alpha} L(t)^{1-\alpha} \tag{3.4}$$

where K denotes capital, L denotes labor and $A > 0$.

3.3. Fiscal policy

The government obtains resources from two different sources: financial funds from international aid, denoted by $aid(t)$ (in per capita terms), and tax collection from non-distortionary lump-sum taxes that are a portion of the per capita income. We consider lump-sum taxes to exclude the standard effects of distortionary taxes from the analysis. Because our paper focuses on the implications of rent-seeking, we prefer to introduce non-distortionary taxes. The reason is that we want to make clear that the mechanisms that we analyze in the model are related to rent-seeking and not to the distortionary effects of taxes, which do not play any role in this paper. Thus:

$$T(t) = \tau y(t) + aid(t) \tag{3.5}$$

where T denotes the per capita revenues and $\tau \in (0, 1)$ is the fixed tax rate.

We denote the ratio international aid-(national) income by $a(t) \equiv aid(t)/y(t)$. Thus, the ratio government revenues-income is as follows:

$$\frac{T(t)}{y(t)} = \tau + a(t) \tag{3.6}$$

In this economy, not all government revenues are devoted to financing the public good[9]. Given that agents may be involved in rent-seeking activities, some public resources are "transferred" to agents[10]. Each individual is endowed with one unit of time each period and decides the portion of time devoted to rent-seeking activities, $l^i_{rs}(t)$,

[9] As aid and corruption are two critical issues for developing economies, many empirical papers have investigated the relationship between both dimensions. Although the studies are not consensual, most of them tend to show that aid enhances corruption (Knack, 2001, Alesina and Weder, 2002). The reason is that aid can generate bad incentives for recipient countries to reduce the need for governments to collect taxes and to encourage rent-seeking activities.

[10] Actually, public officers who manage international aid are the group of agents that can appropriate resources from the government through corruption. However, in our representative agent model, we are considering that the representative agent may devote a fraction of her time to rent-seeking. Alternatively, we might assume that a fraction of the households' members are corrupt officers involved in rent-seeking activities.

and the part devoted to work, $1-l_{rs}^{i}(t)$. The share of government revenues appropriated by rent-seeking activities is an increasing function of the per capita rent-seeking effort:

$$tr(t) = Bl_{rs}(t)^{\beta}T(t) \tag{3.7}$$

where $tr(t)$ denotes the per capita amount of transfers and $l_{rs}(t) = \int_0^1 l_{rs}^j(t)dj$ denotes the per capita amount of rent-seeking efforts; $B \in [0,1]$, $\beta \in (0,1)$. Parameter B is a measure of the productivity of rent-seeking technology, which may be interpreted as an index of institutional weakness. The share of per capita transfers that each agent obtains increases with the rent-seeking effort that the agent makes, l_{rs}^i, and decreases with the other agents rent-seeking efforts, l_{rs}^j:

$$tr^i(t) = \frac{\left(l_{rs}^i(t)\right)^{\theta}}{\int_0^1 \left(l_{rs}^j(t)\right)^{\theta} dj} tr(t) \tag{3.8}$$

where $\theta \in (0,1)$.

The government uses the part of public revenues that
not grabbed by rent-seeking activities to provide the public good:

$$T(t) = tr(t) + g(t) \Leftrightarrow g(t) = T(t) - tr(t) = T(t)\left[1 - Bl_{rs}(t)^{\beta}\right] \tag{3.9}$$

3.4. Firms

Firms maximize profits:

$$\max_{K(t),L(t)} Ag(t)^{1-\alpha}K(t)^{\alpha}L(t)^{1-\alpha} - w(t)L(t) - (\delta + r(t))K(t)$$

where w and r denote the prices of both labor and physical capital, respectively. The first order conditions are standard ones:

$$(1-\alpha)\frac{Ag(t)^{1-\alpha}K(t)^{\alpha}L(t)^{1-\alpha}}{L(t)} = w(t) \tag{3.10}$$

$$\alpha\frac{Ag(t)^{1-\alpha}K(t)^{\alpha}L(t)^{1-\alpha}}{K(t)} = (\delta + r(t)) \tag{3.11}$$

Using the labor market clearing condition $L(t) = (1-l_{rs}(t))N(t)$ in the above equations it yields:

$$(1-\alpha)\frac{y(t)}{(1-l_{rs}(t))} = w(t) \tag{3.12}$$

$$\alpha\frac{y(t)}{k(t)} = (\delta + r(t)) \tag{3.13}$$

3.5. Households

Households face the following optimization problem:

$$\underset{c(t),l_{rs}^{i}(t)}{Max} \int_{t=0}^{\infty} u(c(t))e^{-\rho t}dt \tag{3.14}$$

$$\text{s.t. } r(t)b(t) + w(t)\left(1\text{-}l_{rs}^{i}(t)\right) + \frac{\left(l_{rs}^{i}(t)\right)^{\theta}}{\int_0^1 \left(l_{rs}^{j}(t)\right)^{\theta} dj} tr(t) - \tau y(t) \;=\; \dot{b}(t) + c(t)$$

where $b(t)$ denotes the household assets at time t, that is, the household wealth, and $1 - l_{rs}^{i}(t)$ denotes the amount of time devoted to work in the labor market. The first order conditions associated with the household's optimization problem (3.14) are the following:

$$w(t) \;=\; \frac{\theta \, tr(t)}{(l_{rs}^{i}(t))^{1-\theta} \int_0^1 \left(l_{rs}^{j}(t)\right)^{\theta} dj} \tag{3.15}$$

$$\frac{\dot{c}(t)}{c(t)} \;=\; \frac{1}{\sigma}\left(r(t) - \rho\right) \tag{3.16}$$

$$\lim_{t\to\infty} e^{-\rho t} c(t)^{-\sigma} b(t) \;=\; 0 \tag{3.17}$$

Equation (3.15) means that the marginal income from working should be equal to the marginal income from rent seeking. The second equation (3.16) is the typical Euler equation and the last one (3.17) is the transversality condition. Given that all agents are alike in equilibrium, the time devoted to rent-seeking activity, l_{rs}^{i}, is the same for all agents, $l_{rs}^{i}(t) = l_{rs}^{j}(t) = l_{rs}(t) \; \forall i, j$. This symmetry condition together with equations (3.12), (3.7) and (3.15) imply:

$$\frac{l_{rs}^{1-\beta}}{1 - l_{rs}} \;=\; \frac{\theta B}{(1-\alpha)} \frac{T}{y} \tag{3.18}$$

Using the Implicit Function Theorem on the above equation (3.18), we define $l_{rs}(.)$ as a function that relates the time devoted to rent-seeking activities with the ratio government revenues-income, $\frac{T}{y}$, the productivity of the rent-seeking technology (or index of institutional weakness), B, and the labor share, $(1-\alpha)$:

$$l_{rs}(\underset{+}{\frac{T}{y}}, \underset{+}{B}, \underset{-}{(1-\alpha)})$$

where the signs behind the variables means are the signs of the derivatives (see appendix for details). When either the ratio government revenues-income, $\frac{T}{y}$, or the index of institutional weakness, B, increase, rent-seeking becomes more profitable, encouraging agents to devote more time to rent-seeking activities. To the contrary, a rise in the labor share, $(1-\alpha)$, increases the return of working in the market and so, the opportunity cost of devoting time to rent-seeking, thus discouraging rent-seeking. In order to make the notation more compact, we will re-write the function $l_{rs}\left(\frac{T}{y}, B, (1-\alpha)\right)$ as $l_{rs}\left(\frac{T}{y}\right)$ when this does not cause confusion.

4. Balanced growth path

Using equations (3.4), (3.13), (3.16) and (3.18) we obtain the growth rate of the economy, denoted by v, which is constant when the ratio international aid-income, a, is constant:

$$v = \frac{\dot{c}}{c} = \frac{1}{\sigma}\left(\alpha A^{\frac{1}{\alpha}}\left[\frac{(1-\alpha)}{\theta B}\left(l_{rs}\left(\frac{T}{y}\right)\right)^{1-\beta}\left[1-B\left(l_{rs}\left(\frac{T}{y}\right)\right)^{\beta}\right]\right]^{\frac{1-\alpha}{\alpha}} - \delta - \rho\right) \tag{4.1}$$

Consider that $B > 1-\beta$, then the growth rate reaches its maximum level at the following value (see appendix 8.2 for details)[11]:

$$\left(\frac{T}{y}\right)^{*} = (\tau + a)^{*} = \frac{(1-\alpha)}{\theta}\frac{(1-\beta)^{\frac{1-\beta}{\beta}}}{(B)^{\frac{1}{\beta}} - (1-\beta)^{\frac{1}{\beta}}} \tag{4.2}$$

Thus, the relationship between the growth rate and the ratio of government revenues-income shows a hump-shaped form: first, it is strictly increasing, and then, strictly decreasing (see Figure 4.1). Assuming that $\tau < \left(\frac{T}{y}\right)^{*}$, this implies that the level of international aid over income that maximizes the growth rate, a^{*}, is as follows:

$$a^{*} = \frac{(1-\alpha)}{\theta}\frac{(1-\beta)^{\frac{1-\beta}{\beta}}}{(B)^{\frac{1}{\beta}} - (1-\beta)^{\frac{1}{\beta}}} - \tau \tag{4.3}$$

Notice that the reason for this hump-shaped relation is different from Barro's model (Barro 1990). In Barro's model, the hump-shaped relationship between growth and tax rate is due to the distortionary effect of the income tax on the present-future consumption decision. Moreover, international aid's impact on growth would always be positive, given that the increase in aid would imply a rise in the government's expenditure for the same tax rate. The distortionary tax effect of Barro's model has been erased from

[11] If $B \leq 1-\beta$, the rent-seeking technology would not be productive enough to encourage individuals to devote time to rent-seeking and so, to undermine the benefits of international aid. Consequently, aid always would generate more growth and a strong positive relationship between aid and growth would emerge, which is inconsistent with the empirical evidence.

our model to make it clear that this distortion does not play any role. In our model, an increase in government revenue produces three different effects. First, it increases, before rent-seeking occurs, government expenditure on the public good, which increases the productivity of the private sector and the growth rate (that is the standard effect as in Barro's model)[12]. Second, the increase in government revenues encourages rent-seeking and, thus, reduces the portion of government revenues devoted to productive government expenditure, reducing growth. Third, because agents devote more effort to rent-seeking activities, the labor supply goes down, reducing growth. When the amount of international aid is below level a^*, the first effect prevails, and international aid positively affects growth. When international aid exceeds level a^*, adverse effects predominate, and international aid hampers growth. Thus, an increase in international aid does not always promote growth. Although aid contributes to financing a productive public good, it also encourages rent-seeking. Indeed, if international aid is above threshold a^*, an increase in international aid becomes harmful to growth.

A salient feature of the optimal aid a^* (see 4.3) is that this optimal level increases with the institutional quality (it is a decreasing function of B). Thus, aid is more likely to have counterproductive effects in countries with weaker institutions. This argument is developed further in this section.

Figure 4.1 represents the effect of an improvement in the quality of institutions, that is, a reduction in institutional weakness, B, on growth (see appendix 8.2 for technical details). An improvement in institutional quality reduces the incentive to devote time to rent-seeking, increasing the share of government revenues dedicated to financing the productive public good and the time devoted to work in the market. As a result, it has a positive effect on the growth rate.

Furthermore, an improvement in institutional quality reduces "the rent-seeking promoting effect" of international aid, raising the "optimal" level of international aid that maximizes the growth rate, a^* (see equation 4.3 and figure 4.1). A possible reinterpretation of this result of the model is that, given an amount of aid a, there is a certain threshold level of institutional weakness, $B(a)$, such that the effect of aid on growth is positive if the institutional weakness is lower than this threshold level, $B(a)$:

$$\frac{\partial v(\tau + a, B)}{\partial a} \geq 0 \Leftrightarrow B \leq B(a)$$

[12]When there are no public revenues, $\tau + a = 0$, there is no public good, which is indispensable to produce. Consequently, production is zero, and the gross return on saving is also zero. Therefore, the growth rate is always negative. More precisely, the net return on savings is equal to -δ, and the consumption growth rate is equal to $-\frac{1}{\sigma}(\delta + \rho)$. Barro's model also has this characteristic.

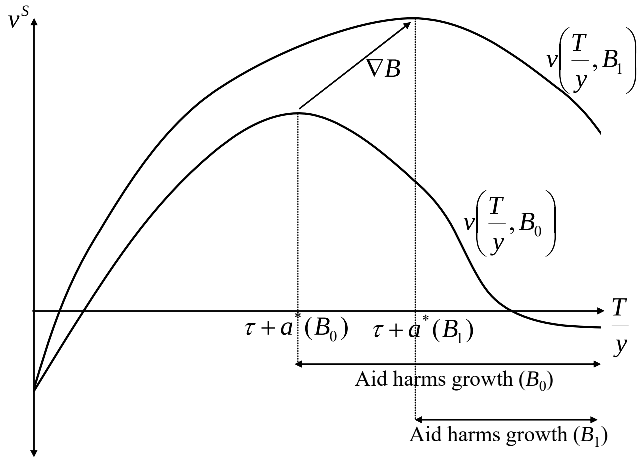

Figure 4.1: Effect of an improvement in institutions ($B_0 > B_1$)

where $B(a)$ is defined as follows:

$$B(a) \overset{def}{\Leftrightarrow} a = a^* = \frac{(1-\alpha)}{\theta} \frac{(1-\beta)^{\frac{1-\beta}{\beta}}}{(B(a))^{\frac{1}{\beta}} - (1-\beta)^{\frac{1}{\beta}}} - \tau \Leftrightarrow$$

$$B(a) = (1-\beta)\left[\frac{(1-\alpha)}{\theta(1-\beta)(a+\tau)} + 1\right]^{\beta}$$

Summarizing, countries with the worst institutions are not only the ones that grow slower, but they are also the ones in which international aid is more likely to be counterproductive[13].

5. Convergence: Two-different countries model

In this section, we study the convergence of the country that receives international aid, which we call *South,* to the country that gives it, which we call *North.* We will analyze how international aid affects the convergence in per capita income.

[13] This result is consistent with empirical findings in the literature, which evidences that the institutional quality of the donor's government is a crucial factor in explaining aid effectiveness. See Ravallion (2014) for a detailed discussion.

5.1. Balanced Growth Path

We assume that there are two countries, *North* and *South*. We will consider in this section that the growth rate in the *North* is exogenous. In fact, the only parameter related to the *North* is the growth rate, v^N, which will have the subscript N, while the growth rate in the South, v^S, will have subscript S[14]. Remaining parameters are referred to the *South*. We assume that:

1. In the absence of international aid, the *North* grows faster than the *South*:

$$v^N > v^S_{\text{no aid}} = \frac{1}{\sigma}\left(\alpha A^{\frac{1}{\alpha}}\left[\frac{(1\text{-}\alpha)}{\theta B}\left(l_{rs}\left(\tau\right)\right)^{1\text{-}\beta}\left[1\text{-}B\left(l_{rs}\left(\tau\right)\right)^{\beta}\right]\right]^{\frac{1\text{-}\alpha}{\alpha}} - \delta - \rho\right) \tag{5.1}$$

2. The growth rate in the *North* is lower than the maximum growth rate in the *South* (see appendix 8.2):

$$v^N < \max_{\frac{T}{y}} v^S = \frac{1}{\sigma}\left(\alpha A^{\frac{1}{\alpha}}\left[\frac{(1\text{-}\alpha)}{\theta B^{\frac{1}{\beta}}}\left(1\text{-}\beta\right)^{\frac{1\text{-}\beta}{\beta}}\beta\right]^{\frac{1\text{-}\alpha}{\alpha}} - \delta - \rho\right) \tag{5.2}$$

3. If all the labor force is devoted to rent-seeking in the *South*, its growth rate would be lower than in the *North*[15]:

$$\begin{aligned} v^N \quad > \quad & \lim_{l_{rs}\to 1}\frac{1}{\sigma}\left(\alpha A^{\frac{1}{\alpha}}\left[\frac{(1\text{-}\alpha)}{\theta B}\left(l_{rs}\right)^{1\text{-}\beta}\left[1\text{-}B\left(l_{rs}\right)^{\beta}\right]\right]^{\frac{1\text{-}\alpha}{\alpha}} - \delta - \rho\right) = \\ & \frac{1}{\sigma}\left(\alpha A^{\frac{1}{\alpha}}\left[\frac{(1\text{-}\alpha)}{\theta B}\left[1\text{-}B\right]\right]^{\frac{1\text{-}\alpha}{\alpha}} - \delta - \rho\right) \end{aligned} \tag{5.3}$$

A possible reason for the *North* growing faster than the *South* is that the *North* may have better institutions (a lower B) than the *South*. Indeed, even if the *North* and the *South* have the same technology and preferences, a difference in institutional quality (in parameter B) would generate differences in growth rates.

We consider that the *North* spends a fraction ψ of its income on international aid to the *South*:

$$aid = \psi y^N(t) \Rightarrow a = \frac{aid}{y^S(t)} = \psi\frac{y^N(t)}{y^S(t)} = \frac{\psi}{\widetilde{y}(t)}$$

[14] In the next section, we will consider the growth rate in the *North* endogenous.

[15] Obviously, this can never happen in autarchy. However, when there is international aid and the ratio international aid-income in the *South* goes to infinity, the amount of per capita labor devoted to rent-seeking goes to one.

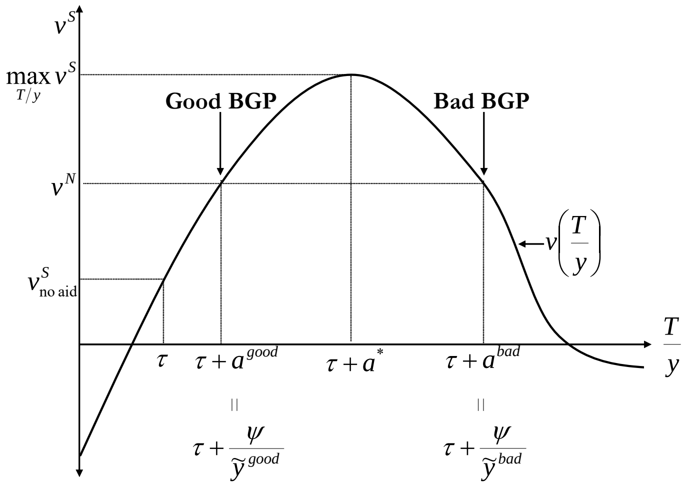

Figure 5.1: Two BGPs: good BGP and bad BGP

where $\widetilde{y}(t) = y^S(t)/y^N(t)$ denotes the convergence index. Thus, along the balanced growth path the convergence index remains constant, $\widetilde{y}(t) = \widetilde{y}\ \forall t$, which implies that both countries are growing at the same rate:

$$v^N = \frac{1}{\sigma}\left(\alpha A^{\frac{1}{\alpha}}\left[\frac{(1\text{-}\alpha)}{\theta B}\left(l_{rs}\left(\tau+\frac{\psi}{\widetilde{y}}\right)\right)^{1-\beta}\left[1 - B\left(l_{rs}\left(\tau+\frac{\psi}{\widetilde{y}}\right)\right)^{\beta}\right]\right]^{\frac{1-\alpha}{\alpha}} - \delta - \rho\right)$$

As Figure 5.1 shows, the hump-shaped relationship between growth and aid implies the existence of two balanced growth paths in the *South* (BGP from now on). In one of the BGPs, the ratio of international aid to income is lower than the optimal level, while in the other, the opposite occurs. Notice that the convergence index is higher in the BGP with the lower ratio of international aid to income than in the other. We will call from now on the "good" balanced growth path (good BGP), the balanced growth path with the higher convergence index, and the "bad" balanced growth path (bad BGP) the other:

$$\widetilde{y}^{good} > \widetilde{y}^{bad} \Longleftrightarrow \frac{\psi}{\widetilde{y}^{good}} = a^{good} < a^* < a^{bad} = \frac{\psi}{\widetilde{y}^{bad}}$$

A simple way to make the *South* economy converge to the good BGP (the one with the higher convergence index) would consist of reducing the international aid that the

South receives. More precisely, the bad BGP disappears if international aid is bounded above by the growth maximizing international aid level, a^*:

$$a(t) = \min\left\{\frac{\psi}{\widetilde{y}}, a^*\right\}$$

Thus, given that $a^* < a^{bad}$, the *South* will converge to the good BGP, where the convergence index is the largest. We will study the dynamics to the BGP in the next subsection.

5.2. Transitional dynamic to the Balanced Growth Path

We will analyze the transitional dynamic to the BGP in two cases:

1. When international aid that the *North* provides to the *South* is proportional to the income of the *North*:

$$aid(t) = \psi y^N(t) \Leftrightarrow a(t) = \frac{aid(t)}{y^S(t)} = \psi \frac{y^N(t)}{y^S(t)} = \frac{\psi}{\widetilde{y}(t)} \tag{5.4}$$

2. When international aid that the *North* provides to the *South* is proportional to the income of the *North,* but it is bounded by the amount of international aid that maximizes the growth rate:

$$a(t) = \min\left\{\frac{\psi}{\widetilde{y}(t)}, a^*\right\} \Leftrightarrow aid(t) = \min\left\{\psi y^N(t), a^* y^S(t)\right\}$$

5.2.1. International aid proportional to the income of the *North*

Let's define $\widetilde{y}(t) = y^S(t)/y^N(t)$, $\widetilde{c}(t) = c^S(t)/y^N(t)$, $\widetilde{k}(t) = k^S(t)/y^N(t)$. The dynamic system which describes the behavior of the economy in the *South* in the case that the received aid is proportional to the income of the *North* would be as follows (see appendix 8.3 for technical details):

$$\frac{\dot{\widetilde{c}}(t)}{\widetilde{c}(t)} = \frac{1}{\sigma}\left(\alpha \frac{\widetilde{y}\left(\widetilde{k}(t)\right)}{\widetilde{k}(t)} - \delta - \rho\right) - v^N \tag{5.5}$$

$$\dot{\widetilde{k}}(t) = \widetilde{y}\left(\widetilde{k}(t)\right)(1\text{-}\tau) + B\left(\widehat{l}_{rs}\left(\widetilde{k}(t)\right)\right)^{\beta}\left[\tau \widetilde{y}\left(\widetilde{k}(t)\right) + \psi\right] \text{-}(\delta + v^N)\widetilde{k}(t)\text{-}\widetilde{c}(t) \tag{5.6}$$

where:

$$\begin{aligned}\widetilde{y}\left(\widetilde{k}(t)\right) &= A^{\frac{1}{\alpha}}\left[\frac{(1\text{-}\alpha)}{\theta B}\left(\widehat{l}_{rs}\left(\widetilde{k}(t)\right)\right)^{1-\beta}\left[1\text{-}B\left(\widehat{l}_{rs}\left(\widetilde{k}(t)\right)\right)^{\beta}\right]\right]^{\frac{1-\alpha}{\alpha}}\widetilde{k}(t)\\
\widehat{l}_{rs}\left(\widetilde{k}(t)\right) &\overset{Def}{\Leftrightarrow} \frac{\left(\widehat{l}_{rs}\left(\widetilde{k}(t)\right)\right)^{1-\beta}}{1-\widehat{l}_{rs}\left(\widetilde{k}(t)\right)} = \frac{\theta B}{(1-\alpha)}\left[\tau+\frac{\psi}{\widetilde{y}\left(\widetilde{k}(t)\right)}\right].\end{aligned}$$

Equation (5.5) is the typical Euler equation and equation (5.6) shows that households have two sources of income: disposable (legal) income, $\widetilde{y}(t)(1\text{-}\tau)$ and, rent-seeking income, $B(l_{rs})^{\beta}[\tau\widetilde{y}(t)+\psi]$.

Lemma 5.1. *There exists $\overline{B} \in \left(\frac{1-\beta}{1-\beta\alpha}, 1\right]$ such*[16] *that $\forall B < \overline{B}$ the function $\widehat{l}_{rs}\left(\widetilde{k}(t)\right)$ exists and is strictly decreasing.*

We assume that $B < \overline{B}$ in order to guarantee the existence of the function $\widehat{l}_{rs}\left(\widetilde{k}(t)\right)$. When $\widehat{l}_{rs}(.)$ exists, it is a strictly decreasing function[17]. Thus, along the transition, time devoted to rent-seeking activities decreases when the capital in the *South* grows faster than in the *North* (i.e., $\widetilde{k}(t) = k^S(t)/y^N(t)$ increases). This feature is because the international aid-wage ratio drops when the *South* grows faster than the *North*. Thus, the reward for rent-seeking, which increases with international aid, grows slower than the reward for working, the wage. Consequently, agents have less incentive to devote time to rent-seeking activities, stimulating work in production.

Figure 5.2 displays the Phase diagram associated with the above dynamic system. While the good BGP is always a saddle point with a unique converging equilibrium path, the bad BGP is either a focus or a source (see appendix 8.6). Multiple equilibria arise when the bad BGP is a source, and some equilibrium paths converge to the bad equilibrium path. Furthermore, the grey line shows the path that would converge to the trivial BGP characterized by zero capital and zero production. However, notice that

[16] Note that $\frac{1-\beta}{1-\beta\alpha} > 1-\beta$ since $\alpha < 1$. Thus, the subset of parameters which satisfy simultaneously this assumption and the previous assumption, $B > 1-\beta$, is non-empty.

[17] When $\overline{B} < B \leq 1$, there are "static" multiple equilibria for low capital levels. That is, there is more than one equilibrium for a given amount of capital. Such multiple equilibria are not related to the dynamic around the BGP. These static multiple equilibria arise due to the following feedback mechanism. If a large amount of labor is devoted to rent-seeking, revenues dedicated to the public good are low, making the marginal productivity of labor and wages low. These low wages encourage rent-seeking. This feedback process may generate a vicious or virtuous circle, which explains this static multiple equilibria.

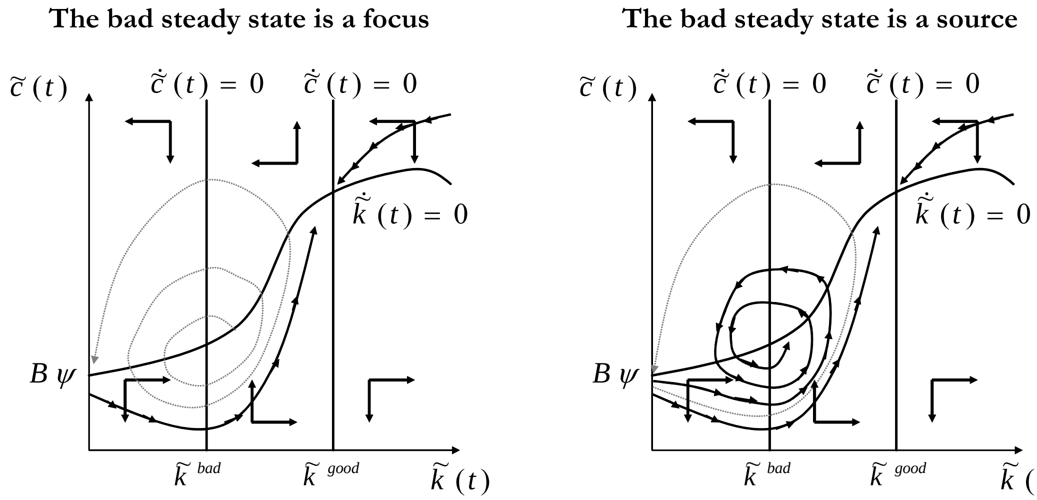

Figure 5.2: Transitional Dynamic to the BGP when aid is proportional to the income in the *North*

consumption would be positive on the trivial BGP[18] because households would consume from international aid, devoting all their time to rent-seeking[19].

5.2.2. Bounded international aid

We now consider that the ratio of international aid income is bounded by the level that maximizes the growth rate in the South, a^*:

$$a(t) = \min\left\{\frac{\psi}{\widetilde{y}(t)}, a^*\right\} \Leftrightarrow aid(t) = \min\left\{\psi y^N(t), a^* y^S(t)\right\}$$

[18] The trivial BGP exists if the return on savings when $\widetilde{k}$ goes to zero is lower than the discount rate of the utility:

$$\lim_{\widetilde{k}\to 0} r\left(\widetilde{k}\right) = \alpha A^{\frac{1}{\alpha}}\left[\frac{(1\text{-}\alpha)}{\theta B}(1\text{-}B)\right]^{\frac{1-\alpha}{\alpha}} - \delta \le \rho$$

This condition means that households would contract debt if they could do it at the equilibrium interest rate, but they cannot since our model has no international capital market. Thus, households neither save nor incur debts at the trivial BGP. Instead, they consume all their resources, which implies that their capital goes to zero. Consequently, production is zero, and households would consume from the rent-seeking activities of international aid.

[19] When the BGP is a focus, results are similar to when it is a source. The difference is that there is not any path that converges to the bad BGP. Instead, paths only converge to the trivial balanced growth path or the good BGP.

The dynamic system which describes the behavior of the economy in the *South* in this case would be as follows:

$$\frac{\dot{\widetilde{c}}(t)}{\widetilde{c}(t)} = \frac{1}{\sigma}\left(\alpha\frac{\widetilde{y}\left(\widetilde{k}(t)\right)}{\widetilde{k}(t)} - \delta - \rho\right) - v^N \tag{5.7}$$

$$\dot{\widetilde{k}}(t) = \widetilde{y}\left(k(t)\right)(1-\tau) + B\left(\widehat{l}_{rs}\left(\widetilde{k}(t)\right)\right)^{\beta}\left[\tau\widetilde{y}\left(k(t)\right) + \min\left\{\psi, a^*\widetilde{y}\left(\widetilde{k}(t)\right)\right\}\right] - (\delta + v^N)\widetilde{k}(t) - \widetilde{c}(t) \tag{5.8}$$

where:

$$\widetilde{y}\left(\widetilde{k}(t)\right) = A^{\frac{1}{\alpha}}\left[\frac{(1\text{-}\alpha)}{\theta B}\left(\widehat{l}_{rs}\left(\widetilde{k}(t)\right)\right)^{1-\beta}\left[1\text{-}B\left(\widehat{l}_{rs}\left(\widetilde{k}(t)\right)\right)^{\beta}\right]\right]^{\frac{1-\alpha}{\alpha}}\widetilde{k}(t)$$

$$\widehat{l}_{rs}\left(\widetilde{k}(t)\right) \overset{Def}{\Leftrightarrow} \frac{\left(\widehat{l}_{rs}\left(\widetilde{k}(t)\right)\right)^{1-\beta}}{1-\widehat{l}_{rs}\left(\widetilde{k}(t)\right)} = \frac{\theta B}{(1-\alpha)}\left[\tau + \min\left\{\frac{\psi}{\widetilde{y}(t)}, a^*\right\}\right]$$

The critical difference between this dynamic system and the one described in subsection 5.2.1 (unbounded aid) is that now the ratio of government revenues-income in the *South* (T/y) is bounded above by $\tau + a^*$. Since the amount of time devoted to rent-seeking (an increasing function of T/y) is also bounded above by the threshold l_{rs}^* defined as follows:

$$l_{rs}^* \overset{Def}{\Leftrightarrow} \frac{(l_{rs}^*)^{1-\beta}}{1-l_{rs}^*} = \frac{\theta B}{(1-\alpha)}\left[\tau + a^*\right]$$

Since aid is bounded above by a^*, the right part of the hump-shaped curve that relates growth in the *South* to aid disappears (see Figure 5.1). Since the curve that relates growth in the South and aid is strictly increasing in the interval $[0, a^*]$, it crosses with the North's growth rate just once. As a consequence, there is a unique BGP and a unique equilibrium path converging to it (see the appendix 8.7).

As we have emphasized before, the optimal aid a^* (see 4.3) increases with the institutional quality (it is a decreasing function of B). Thus, aid is more likely to have counterproductive effects in countries with weaker institutions. Consequently, if aid policy aims to make countries converge to the good BGP, aid should be less generous in countries with weaker institutions. This result may imply a paradox: countries with weaker institutions and lower per capita income should receive less international aid; otherwise, the rent-seeking promoting effect would harm those most in need.

5.3. The role of institutions in deterring bad BGP

In this subsection, we analyze the role of institutional quality in deterring bad BGP. To do so, we will examine the case in which international aid is not bounded above since it is the case in which multiple BGPs may arise. That is, we assume that $aid(t) = \psi y^N(t)$ (see equation (5.4)).We also remove assumption 3, defined at equation (5.3), since such an assumption guarantees that there are two BGPs, and we want to find under which conditions a unique BGP exists.

Proposition 5.2. *If $B \leq \underline{B}$, then there is a unique BGP. If $B > \underline{B}$, then there are two BGPs, the good and the bad BGPs.*

This proposition shows that the bad BGP only arises when institutions are weak. The incentive to devote time to rent-seeking activities is faint if institutions are strong enough. Consequently, the bad BGP in which agents spend more time rent-seeking does not arise.

6. The case of *two-alike* countries

In previous sections, we investigated the effect of international aid provided by the *North* to the *South*, assuming that the *North* has more potential growth. We go one step forward in this section and wonder what would occur if the *South* and the *North* were identical in all aspects except the initial capital level. Is it possible that two equal countries may converge to two different long-run equilibria? To answer these questions, we will set up a world composed of two countries, the *North* and the *South*, with identical economic features except for the initial capital. Furthermore, we assume that the aid that each country receives is as follows:

$$\begin{aligned} aid^i(t) &= \psi\left(y^j(t) - y^i(t)\right); \qquad i,j \in \{S,N\}\,;\; i \neq j;\; \psi > 0 \\ a^i(t) &= \frac{aid^i(t)}{y^i(t)} = \psi\left(\frac{y^j(t)}{y^i(t)} - 1\right) \end{aligned} \tag{6.1}$$

where[20] $\psi \in (0, \tau)$ and $aid^i(t)$ denotes the aid received by country i, and $a^i(t)$ denotes the ratio of aid received by country i to income at country i. I. If the value of the aid is negative, the country does not receive aid but gives aid to the other country. That is, if the aid of country i is negative ($aid^i(t) < 0$), country i is the donor, and the other country, j, is the receptor country. Note that the sum of the aid of the two countries is always zero. The amount that the receptor country receives is equal to the aid that the donor country gives:

$$\begin{aligned} & aid^S(t) = \psi\left(y^S(t) - y^N(t)\right) = -\psi\left(y^N(t) - y^S(t)\right) = -aid^N(t) \Rightarrow \\ & aid^N(t) + aid^S(t) = 0 \end{aligned}$$

[20] The assumption $\psi < \tau$ guarantees that the aid that the donor country provides to the receptor country is always smaller than the revenues of the donor country: $aid^{receptor}(t) = \psi\left(y^{donor}(t) - y^{receptor}(t)\right) < \tau y^{donor}(t)$. This assumption is needed to satisfy the budget constraint of the donor.

The growth rate on the BGP would be as follows (see equation 4.1):

$$v^i = v\left(\frac{T^i}{y^i}\right) = v\left(\tau + \psi\left(\left(\frac{y^j}{y^i}\right) \text{-}1\right)\right) \qquad i,j \in \{S,N\}\,;\ i \neq j$$

where $v\,(.)$ is defined as the function that relates the growth rate of consumption to the ratio of public revenues to income:

$$v\left(\frac{T}{y}\right) = \frac{1}{\sigma}\left\{\alpha A^{\frac{1}{\alpha}}\left[\frac{(1\text{-}\alpha)}{\theta B}\left(l_{rs}\left(\frac{T}{y}\right)\right)^{1-\beta}\left[1\text{-}B\left(l_{rs}\left(\frac{T}{y}\right)\right)^{\beta}\right]\right]^{\frac{1-\alpha}{\alpha}} - \delta - \rho\right\}$$

where we have used equations (3.6) and (6.1).

To have a BGP, the growth rates of both countries should be the same. Thus, the following condition should hold:

$$\begin{aligned} v^S &= v^N \Leftrightarrow \\ v\left(\tau + \psi\left(\frac{1}{\widetilde{y}} - 1\right)\right) &= v\left(\tau + \psi\left(\widetilde{y} - 1\right)\right) \end{aligned} \tag{6.2}$$

where $\widetilde{y}(t) = y^S(t)/y^N(t)$. It is obvious from equation (6.2) that there is always a BGP where $\widetilde{y} = 1$. We will call such a BGP "symmetric" BGP, given that the incomes of the *South* and the *North* equalize. Any other BGP where the incomes of the *South* and the *North* do not equalize, $\widetilde{y} \neq 1$, will be called "asymmetric" BGP. If there is an asymmetric BGP where, for instance, income in the *South* is lower than in the *North,* $\widetilde{y}^{BGP1} < 1$, "$BGP1$", then there is another asymmetric BGP where income in the South is higher than in the North where $\widetilde{y}^{BGP2} = 1/\widetilde{y}^{BGP1}$, "$BGP2$". That is:

$$v\left(\tau + \psi\left(\frac{1}{\widetilde{y}^{BGP1}} - 1\right)\right) = v\left(\tau + \psi\left(\widetilde{y}^{BGP1} - 1\right)\right) \Leftrightarrow \tag{6.3}$$

$$v\left(\tau + \psi\left(\frac{1}{\widetilde{y}^{BGP1}} - 1\right)\right) = v\left(\tau + \psi\left(\left(\frac{1}{\frac{1}{\widetilde{y}^{BGP1}}}\right) - 1\right)\right) \Leftrightarrow \tag{6.4}$$

$$v\left(\tau + \psi\left(\widetilde{y}^{BGP2} - 1\right)\right) = v\left(\tau + \psi\left(\left(\frac{1}{\widetilde{y}^{BGP2}}\right) - 1\right)\right) \tag{6.5}$$

Therefore, there is always an even number of asymmetric BGPs.

Proposition 6.1. *If $\tau < a^*$,[21] there is $\widehat{B} > 0$ such that if $B > \widehat{B}$, there are at least three BGPs, with only one being symmetric, $\widetilde{y} = 1$. Moreover, the growth rate along the symmetric BGP is higher than in any asymmetric BGPs.*

[21] When $\tau > a^*$, it is also possible to have asymmetric BGP, as we show in the appendix.

Thus, when institutions are weak enough, asymmetric BGPs arise. In such asymmetric BGPs, the rent-seeking effect on the receptor country is strong enough to impede the convergence of the receptor country to the donor country. Moreover, aid also undermines the growth rate in the donor country since aid deviates public revenues from productive public goods to international aid. Thus, due to the rent-seeking effect of aid in the receptor country and the deviation of public resources to aid in the donor country, the growth rate in the asymmetric BGPs are lower than in the symmetric one.

Figure 6.1 illustrates proposition 6.1. We plotted the curve that relates the growth rate of the *South* on the BGP with the convergence index $\widetilde{y}$ and the curve that relates the growth rate of the *North* on the BGP with the convergence index $\widetilde{y}$. On the BGP, the *South* and the *North* should grow at the same rate (see equation 6.2).Thus, when curves representing the North and South growth cross, both countries grow at the same rate. Consequently, cross points represent BGPs. The two curves cross three times: once at the symmetric BGP with convergence, $\widetilde{y}^{symmetric\ BGP} = 1$, and twice at two BGPs where there is no convergence. In one asymmetric BGPs, the South is poorer than the North, $\widetilde{y}^{asymmetric,\ BGP1} < 1$, and the opposite occurs in the other, $\widetilde{y}^{asymmetric,\ BGP2} > 1$. The two asymmetric BGPs imply the same growth rates. They are symmetric between them in the following sense: $\widetilde{y}^{asymmetric,\ BGP1} = 1/\widetilde{y}^{asymmetric,\ BGP2}$ (see equations 6.3 to 6.5). Thus, we show that two identical countries may end up not converging to the same levels of per capita income simply because of the existence of international aid and the fact that they start with different initial levels of per capita capital. In this sense, we can state that history matters.

The growth rates along asymmetric BGPs (with no convergence and international aid) are lower than along the symmetric BGP (with convergence in per capita income and no international aid). The intuition of this result emerges from Figure 6.1. In the asymmetric BGP, the receptor country (the *South* in the first asymmetric BGP and the *North* in the second one) receives an amount of international aid higher than $a^*(a^S > a^*$ in the first asymmetric BGP and $a^N > a^*$ in the second one). Therefore, the effect of international aid on promoting rent-seeking is stronger than the effect of providing more productive public goods in the receptor country. Consequently, international aid is harming growth in the receptor country. On the other hand, the donor country (the North in the first asymmetric BGP and the South in the second) has a tax rate and a ratio of revenues to income lower than a^*. Therefore, the donor country, conversely to the receptor one, is in the increasing part of the curve that relates growth with public revenues (displayed in 4.1). Thus, an increase in the ratio of government revenue to income in the donor country would increase the growth rate. That is, in the case of the donor country, the effect of government revenues on the productive public good is stronger than the effect on rent-seeking. Thus, international aid, which reduces the net revenues of the donor country, also harms the growth rate of the donor country.

Summarizing, on the asymmetric BGP, international aid promotes rent-seeking in

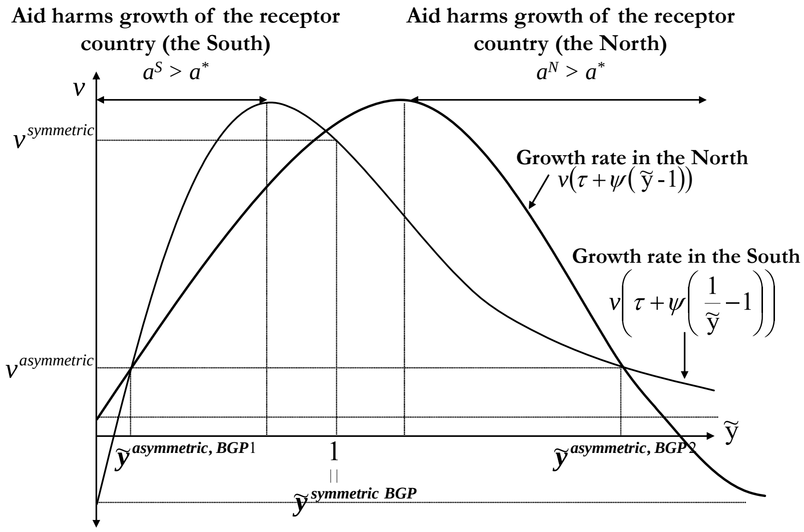

Figure 6.1: Relationship between growth and international aid in the *two-alike* countries case

the receptor country and deviates resources from productive public investment to international aid in the donor country, reducing growth rates in both countries. Thus, when institutions are not strong enough, international aid would harm the cross-country convergence in per capita income and the growth rate of the world economy.

7. Conclusions

Recent empirical evidence on foreign aid and economic growth suggests a hump-shaped relation between these variables. This paper has analyzed the relationship between aid and growth in a context in which rent-seeking activities erode the effort of governments to provide a public good that generates growth. We built a model in which agents devote time to work and rent-seeking activities to appropriate public revenues from non-distortionary taxes and foreign aid. Furthermore, the government uses the revenues after rent-seeking to finance a public good, which generates growth.

We showed that the relationship between aid and growth is hump-shaped. When aid increases, three effects on growth appear: (i) aid raises the government's resources and so the provision of the public good, increasing the productivity of the private sector and the growth rate; (ii) the increase in the government's resources raises the profitability of rent-seeking activities, reducing public revenues after rent-seeking and so reducing growth; (iii) because agents devote more time to rent-seeking activities, labor supply drops, reducing growth. We proved that the first effect prevails for low aid levels, whereas the other two are stronger for high aid levels.

Institutions play a crucial role in the impact of aid in growth. Aid is more likely to have a counterproductive effect on growth in countries with bad institutions since aid's encouraging effect on rent-seeking is stronger. Thus, institutional improvements are fundamental to reducing international aid's negative impact on promoting rent-seeking. Furthermore, we proved that there exists a minimum level of institutional quality such that aid generates positive effects on growth. Above this level, aid would harm growth. In this case, aid should be reduced to a more appropriate amount to avoid such undesirable effect.

We examined the question of the convergence of the receptor country to the donor country. We first analyzed the case where the receptor and the donor country are different. More precisely, the donor country has a larger potential long-run growth than the receptor country in the absence of international aid. We assumed that international aid is a fraction of the donor's income. We showed that two balanced growth paths (BGPs) emerge with different convergence levels. The degree of convergence is measured as the ratio between per capita income in the receptor and donor countries. We called the BGP with the higher convergence degree "good BGP" and the other "bad BGP". We demonstrated that a drop in the amount of international aid might increase the per capita income convergence of the receptor country to the donor.

More precisely, if the amount of aid per income of the receptor country is bounded above by the level of aid that maximizes the growth rate, the receptor country will always converge to the good BGP since the bad BGP disappears. Furthermore, the maximizing growth threshold of aid decreases with institutional weakness. Thus, if the objective of aid policy is to foster convergence, a paradox arises. Those countries with worse institutions are poorer but should receive less aid. Since the rent-seeking promoting effect of aid is higher in those countries and more likely to erode their convergence with the donor countries. To help these countries with aid, a previous step should be undertaken: institutional reforms.

Furthermore, we showed that an improvement in the institutional quality of the receptor country not only increases the convergence of the receptor to the donor country in the good BGP but also makes the donor country converge to the good BGP. The reason is that the bad BGP disappears when the institutional improvement is high enough.

We then analyzed the transition dynamic to the BGP. When the initial ratio between the receptor country's per capita income and the donor's per capita income is smaller than the value of the BGP, the transitional dynamic involves a decline in the time devoted to rent-seeking activities. The reason is that the reward for working (the wage) increases faster than the reward for rent-seeking (tax collection plus international aid received). Furthermore, multiple equilibria may arise along the transitional dynamic.

Finally, we raised the question of whether international aid may prevent the convergence of totally identical countries except for their initial amount of capital. In this setting of two-alike countries, the richer country donates international aid to the other. We demonstrated that international aid might prevent the convergence of both countries to the same BGP. Indeed, international aid may generate asymmetric BGPs with no convergence in per capita income when institutions are not strong enough. When asymmetric BGPs arise, international aid from the richer to the poorer countries self-perpetuates. Furthermore, the growth rate in these asymmetric BGPs, without convergence in per capita income, is lower than the growth rate of the symmetric BGP, in which countries converge in per capita income. This last result is because aid promotes rent-seeking in the receptor country and deviates resources from productive public investment to international aid in the donor country, harming growth in both countries. Thus, we showed that aid is bad for growth and convergence when institutions are weak. So, we reached the same conclusion as before: institutional quality should be improved so that aid is effective for growth and convergence.

8. Appendix

8.1. Time devoted to rent-seeking activities:

Using the Implicit Function Theorem and equation (3.18), we define $l_{rs}(.)$ as a function that relates the time devoted to rent-seeking activities with the ratio government revenues-income, $\frac{T}{y}$, the productivity of the rent-seeking technology (or index of institutional weakness), B, and the labor share, $(1-\alpha)$:

$$l_{rs}\left(\frac{T}{y}, B, (1-\alpha)\right) \overset{Def}{\Longleftrightarrow} \frac{\left(l_{rs}\left(\frac{T}{y}, B, (1-\alpha)\right)\right)^{1-\beta}}{1-l_{rs}\left(\frac{T}{y}, B, (1-\alpha)\right)} = \frac{\theta B}{(1-\alpha)}\frac{T}{y} \tag{8.1}$$

where

$$\frac{\partial l_{rs}\left(\frac{T}{y}, B, (1-\alpha)\right)}{\partial\left(\frac{T}{y}\right)} = \frac{\theta B}{(1-\alpha)} \frac{[1-l_{rs}]^2 l_{rs}^{\beta}}{(1-\beta)[1-l_{rs}]+l_{rs}} > 0 \tag{8.2}$$

$$\frac{\partial l_{rs}\left(\frac{T}{y}, B, (1-\alpha)\right)}{\partial B} = \frac{\theta}{(1-\alpha)} \frac{T}{y} \frac{[1-l_{rs}]^2 l_{rs}^{\beta}}{(1-\beta)[1-l_{rs}]+l_{rs}} > 0 \tag{8.3}$$

$$\frac{\partial l_{rs}\left(\frac{T}{y}, B, (1-\alpha)\right)}{\partial (1-\alpha)} = -\frac{\theta B}{(1-\alpha)^2} \frac{T}{y} \frac{[1-l_{rs}]^2 l_{rs}^{\beta}}{(1-\beta)[1-l_{rs}]+l_{rs}} < 0 \tag{8.4}$$

8.2. Growth rate and its maximum level:

Substituting the government budget constraint (equation 3.9) in the production function (equation 3.4) yields:

$$y = A\underbrace{\left[\left(\frac{T}{y}\right)\left[1-Bl_{rs}^{\beta}\right]y\right]}_{g}^{1-\alpha} k^{\alpha}(1-l_{rs})^{1-\alpha} \Leftrightarrow$$

$$y = A^{\frac{1}{\alpha}}\left[\left(\frac{T}{y}\right)\left[1-Bl_{rs}^{\beta}\right](1-l_{rs})\right]^{\frac{1-\alpha}{\alpha}} k$$

If we now substitute equation (3.18), we obtain:

$$y = A^{\frac{1}{\alpha}}\left[\frac{(1-\alpha)}{\theta B} l_{rs}^{1-\beta}\left[1-Bl_{rs}^{\beta}\right]\right]^{\frac{1-\alpha}{\alpha}} k \tag{8.5}$$

Using equations (3.13) and (3.16) it yields:

$$v = \frac{1}{\sigma}\left(\alpha A^{\frac{1}{\alpha}}\left[\frac{(1-\alpha)}{\theta B} l_{rs}^{1-\beta}\left[1-Bl_{rs}^{\beta}\right]\right]^{\frac{1-\alpha}{\alpha}} - \rho - \delta\right)$$

The function reaches the maximum when:

$$\frac{\partial\left[l_{rs}^{1-\beta}\left[1-Bl_{rs}^{\beta}\right]\right]}{\partial l_{rs}} = l_{rs}^{1-\beta}\left[1-Bl_{rs}^{\beta}\right]\left[\frac{(1-\beta)}{l_{rs}} - \frac{\beta B l_{rs}^{\beta-1}}{\left[1-Bl_{rs}^{\beta}\right]}\right] = 0 \Leftrightarrow \tag{8.6}$$

$$(1-\beta)\left[1-Bl_{rs}^{\beta}\right] = \beta B l_{rs}^{\beta} \Leftrightarrow l_{rs} = \left(\frac{1-\beta}{B}\right)^{\frac{1}{\beta}} \Leftrightarrow$$

$$\left(\frac{T}{y}\right)^{*} = \frac{(1-\alpha)}{\theta} \frac{(1-\beta)^{\frac{1-\beta}{\beta}}}{(B)^{\frac{1}{\beta}} - (1-\beta)^{\frac{1}{\beta}}}$$

where we use equation (3.18) in the last equation. It follows from the chain rule that:

$$\frac{\partial v}{\partial\left(\frac{T}{y}\right)} = \underbrace{\frac{\partial v}{\partial\left(l_{rs}^{1-\beta}\left[1-Bl_{rs}^{\beta}\right]\right)}}_{\oplus} \frac{\partial\left[l_{rs}^{1-\beta}\left[1-Bl_{rs}^{\beta}\right]\right]}{\partial l_{rs}} \underbrace{\frac{\partial l_{rs}}{\partial\left(\frac{T}{y}\right)}}_{\oplus}$$

Thus, the sign of the derivative of the growth rate, v, with respect to the ratio government revenues-income, T/y, is the same as the derivative defined in equation (8.6): it is positive for values smaller than $(T/y)^*$ and negative after reaching the maximum at $(T/y)^*$. Thus, there is a hump-shaped relationship between the growth rate and the ratio revenues-income.

The relationship between the growth rate and the index of institutional weakness is given by the following derivative:

$$\frac{\partial\left[\frac{l_{rs}^{1-\beta}\left[1-Bl_{rs}^{\beta}\right]}{B}\right]}{\partial B} = -\frac{l_{rs}^{1-\beta}\left[1-Bl_{rs}^{\beta}\right]}{B} \times$$
$$\left\{\frac{l_{rs}^{\beta}}{\left[1-Bl_{rs}^{\beta}\right]} + \frac{1}{B} + \left[\frac{l_{rs}^{\beta}}{\left[1-Bl_{rs}^{\beta}\right]} - (1-\beta)\frac{1}{l_{rs}}\right] \frac{\partial l_{rs}\left(\frac{T}{y}, B, (1-\alpha)\right)}{\partial B}\right\}$$

Using equations (8.1) and (8.3) it follows that:

$$\frac{\partial l_{rs}\left(\frac{T}{y}, B, (1-\alpha)\right)}{\partial B} = \frac{1}{B}\frac{l_{rs}^{1-\beta}}{1-l_{rs}}\frac{\left[1-l_{rs}\right]^2 l_{rs}^{\beta}}{(1-\beta)\left[1-l_{rs}\right]+l_{rs}} =$$
$$\frac{1}{B}\frac{\left[1-l_{rs}\right] l_{rs}}{(1-\beta)\left[1-l_{rs}\right]+l_{rs}} \Rightarrow$$
$$(1-\beta)\frac{1}{l_{rs}}\frac{\partial l_{rs}\left(\frac{T}{y}, B, (1-\alpha)\right)}{\partial B} = \frac{1}{B}\frac{(1-\beta)\left[1-l_{rs}\right]}{(1-\beta)\left[1-l_{rs}\right]+l_{rs}} < \frac{1}{B} \Rightarrow$$
$$\frac{\partial\left[\frac{l_{rs}^{1-\beta}\left[1-Bl_{rs}^{\beta}\right]}{B}\right]}{\partial B} < -\frac{l_{rs}^{1-\beta}\left[1-Bl_{rs}^{\beta}\right]}{B} \times$$
$$\left\{\frac{l_{rs}^{\beta}}{\left[1-Bl_{rs}^{\beta}\right]} + \frac{1}{B} + \frac{l_{rs}^{\beta}}{\left[1-Bl_{rs}^{\beta}\right]}\frac{\partial l_{rs}\left(\frac{T}{y}, B, (1-\alpha)\right)}{\partial B} - \frac{1}{B}\right\} < 0$$

It follows from the previous equation that:

$$\frac{\partial\left[\frac{l_{rs}^{1-\beta}\left[1-Bl_{rs}^{\beta}\right]}{B}\right]}{\partial B} < 0$$

which implies that:

$$\frac{\partial v}{\partial B} = \underbrace{\frac{\partial v}{\partial\left(l_{rs}^{1-\beta}\left[1-Bl_{rs}^{\beta}\right]\right)}}_{\oplus}\underbrace{\frac{\partial\left[l_{rs}^{1-\beta}\left[1-Bl_{rs}^{\beta}\right]\right]}{\partial B}}_{\ominus} < 0$$

8.3. Transitional dynamics: International aid proportional to the income of the *North*

The Euler Equation for the *South* is as follows (see equation 3.16):

$$\frac{\dot{c}^S(t)}{c^S(t)} = \frac{1}{\sigma}\left(r^S(t)-\rho\right)$$

Because we have defined $\widetilde{c}(t) = c^S(t)/y^N(t)$ and so $\frac{\dot{\widetilde{c}}(t)}{\widetilde{c}(t)} = \frac{\dot{c}^S(t)}{c^S(t)} - v^N$ and using equation (3.13), we obtain the first equation:

$$\frac{\dot{\widetilde{c}}(t)}{\widetilde{c}(t)} = \frac{1}{\sigma}\left(\alpha\frac{\widetilde{y}\left(\widetilde{k}(t)\right)}{\widetilde{k}(t)}-\delta-\rho\right) - v^N$$

Similarly, from the budget constraint of the household's optimization problem at *South* (equation 3.14) and considering equation (3.5), equation (5.4), definition $\widetilde{k}(t) = k^S(t)/y^N(t)$, the clearing condition in the capital market, $k(t) = b(t)$, and the fact that the time devoted to rent-seeking activity, l_{rs}^i, is the same for all agents in the *South*, $l_{rs}^i(t) = l_{rs}^j(t) = l_{rs}(t)$ we obtain:

$$\dot{\widetilde{k}}(t) = r^S(t)\widetilde{k}(t) + w^S(t)\frac{(1-l_{rs}(t))}{y^N(t)} + Bl_{rs}(t)^{\beta}\left[\tau\widetilde{y}\left(k(t)\right)+\psi\right] - v^N\widetilde{k}(t) - \tau\widetilde{y}(t) - \widetilde{c}(t)$$

Then, using equations (3.12), (3.13) and (8.7) we obtain:

$$\dot{\widetilde{k}}(t) = (1-\tau)\,\widetilde{y}(t) + Bl_{rs}(t)^{\beta}\left[\tau\widetilde{y}\left(k(t)\right)+\psi\right] - (\delta+v^N)\widetilde{k}(t) - \widetilde{c}(t)$$

and, from equations (8.5), (5.4) and (3.6) we obtain:

$$\begin{aligned}
\frac{y^S(t)}{y^N(t)} &= A^{\frac{1}{\alpha}}\left[\frac{(1-\alpha)}{\theta B}\left(l_{rs}(t)\right)^{1-\beta}\left[1-B\left(l_{rs}(t)\right)^{\beta}\right]\right]^{\frac{1-\alpha}{\alpha}}\frac{k(t)}{y^N(t)} \Rightarrow \\
\widetilde{y}(t) &= A^{\frac{1}{\alpha}}\left[\frac{(1-\alpha)}{\theta B}\left(l_{rs}(t)\right)^{1-\beta}\left[1-B\left(l_{rs}(t)\right)^{\beta}\right]\right]^{\frac{1-\alpha}{\alpha}}\widetilde{k}(t)
\end{aligned} \tag{8.7}$$

Finally, from equations (3.6), (3.18), (5.4) (8.7), we obtain:

$$\frac{\left(l_{rs}(t)\right)^{1-\beta}}{1-l_{rs}(t)} = \frac{\theta B}{(1-\alpha)}\left(\tau+\frac{\psi}{A^{\frac{1}{\alpha}}\left[\frac{(1-\alpha)}{\theta B}\left(l_{rs}(t)\right)^{1-\beta}\left[1-B\left(l_{rs}(t)\right)^{\beta}\right]\right]^{\frac{1-\alpha}{\alpha}}\widetilde{k}(t)}\right)$$

Thus, the amount of time devoted to work in the market is a function of the ratio capital in the *South* to income in the *North* $\widetilde{k}(t)$:

$$l_{rs}(t) = \widehat{l}_{rs}(\widetilde{k}(t))$$

8.4. Proof of Lemma 5.1

From, appendix 8.3 we know:

$$\begin{aligned}
&\widehat{l}_{rs}(\widetilde{k}) \overset{Def}{\Leftrightarrow} \\
&\frac{\left(\widehat{l}_{rs}(\widetilde{k})\right)^{1\text{-}\beta}}{1-\widehat{l}_{rs}(\widetilde{k})} = \frac{\theta B}{(1\text{-}\alpha)}\left(\tau+\frac{\psi}{A^{\frac{1}{\alpha}}\left[\frac{(1\text{-}\alpha)}{\theta B}\left(\widehat{l}_{rs}(\widetilde{k})\right)^{1\text{-}\beta}\left[1-B\left(\widehat{l}_{rs}(\widetilde{k})\right)^{\beta}\right]\right]^{\frac{1\text{-}\alpha}{\alpha}}\widetilde{k}}\right) \Leftrightarrow \\
&\widetilde{k} = f(l_{rs}) = \frac{\frac{\theta B}{(1-\alpha)}\psi}{A^{\frac{1}{\alpha}}\left(\frac{(l_{rs})^{1-\beta}}{1-l_{rs}}-\frac{\theta B}{(1-\alpha)}\tau\right)\left[\frac{(1-\alpha)}{\theta B}\left(l_{rs}\right)^{1-\beta}\left[1-B\left(l_{rs}\right)^{\beta}\right]\right]^{\frac{1-\alpha}{\alpha}}}
\end{aligned}$$

If the above function $f(l_{rs})$ is invertible, then its inverse function is $\widehat{l}_{rs}(\widetilde{k}) = f^{-1}(\widetilde{k})$. Thus, in order to prove the existence of $\widehat{l}_{rs}(\widetilde{k})$, we will prove that $f(l_{rs})$ is invertible. We concentrate on the relevant range in which $\widetilde{k}$ is positive, that is when $\frac{(l_{rs})^{1-\beta}}{1-l_{rs}} - \frac{\theta B}{(1-\alpha)}\tau > 0 \Leftrightarrow l_{rs} > l_{rs}^{\min}$, where $l_{rs}^{\min}$ is defined as follows:

$$l_{rs}^{\min} \overset{Def}{\Leftrightarrow} \frac{(l_{rs})^{1-\beta}}{1-l_{rs}} = \frac{\theta B}{(1-\alpha)}\tau \tag{8.8}$$

The derivative of $f(l_{rs})$ is as follows

$$\frac{\partial f(l_{rs})}{\partial l_{rs}} = \tag{8.9}$$

$$-\frac{f(l_{rs})}{l_{rs}}\left[\frac{\frac{(l_{rs})^{1\text{-}\beta}}{1-l_{rs}}}{\frac{(l_{rs})^{1\text{-}\beta}}{1-l_{rs}}\text{-}\frac{\theta B}{(1\text{-}\alpha)}\tau}\left[(1\text{-}\beta)+\frac{l_{rs}}{1-l_{rs}}\right]+\frac{1\text{-}\alpha}{\alpha}\left[(1\text{-}\beta)\text{-}\frac{\beta B\,(l_{rs})^{\beta}}{1\text{-}B\,(l_{rs})^{\beta}}\right]\right] <$$

$$-\frac{f(l_{rs})}{l_{rs}}\frac{1}{\alpha}\left[\alpha\frac{l_{rs}}{1-l_{rs}}+(1\text{-}\beta)\text{-}\frac{\beta\,(1\text{-}\alpha)\,B\,(l_{rs})^{\beta}}{1\text{-}B\,(l_{rs})^{\beta}}\right]$$

where in the inequality we used the fact that $\frac{\frac{(l_{rs})^{1-\beta}}{1-l_{rs}}}{\frac{(l_{rs})^{1-\beta}}{1-l_{rs}}-\frac{\theta B}{(1-\alpha)}\tau} > 1$.

The rest of the proof is based on three remarks: Remark 1 proves that the function $f(l_{rs})$ is invertible and continuous, Remark 2 proves that the function $f(l_{rs})$ is strictly decreasing, and Remark 3 proves that $\overline{B} < 1$ when $\frac{1-\alpha}{\alpha} > 1$ (see footnote 4).

Remark 1. $x - \frac{\beta(1-\alpha)B(l_{rs})^{\beta}}{1-B(l_{rs})^{\beta}} \geq 0 \Leftrightarrow l_{rs} \leq \left(\frac{1}{B}\frac{x}{x+\beta(1-\alpha)}\right)^{\frac{1}{\beta}}$

Thus, if $l_{rs} \leq \left(\frac{1}{B}\frac{1-\beta}{1-\beta+\beta(1-\alpha)}\right)^{\frac{1}{\beta}} = \left(\frac{1}{B}\frac{1-\beta}{1-\beta\alpha}\right)^{\frac{1}{\beta}}$ then $(1-\beta) - \frac{\beta(1-\alpha)B(l_{rs})^{\beta}}{1-B(l_{rs})^{\beta}} > 0 \Rightarrow \frac{\partial f(l_{rs})}{\partial l_{rs}} < 0$ (see 8.9).

When $\left(\frac{1}{B}\frac{1-\beta}{1-\beta\alpha}\right)^{\frac{1}{\beta}} > 1 \Leftrightarrow B < \frac{1-\beta}{1-\beta\alpha}$ then $\forall l_{rs} \in \left[l_{rs}^{\min}, 1\right] : \frac{\partial f(l_{rs})}{\partial l_{rs}} < 0$.

Let's define the following function for the case in which $B \geq \frac{1-\beta}{1-\beta\alpha}$:

$$\phi(B) = \max_{l_{rs}\in\left[\left(\frac{1}{B}\frac{1-\beta}{1-\beta\alpha}\right)^{\frac{1}{\beta}},1\right]} \frac{l_{rs}\left(1-B\,(l_{rs})^{\beta}\right)(1-l_{rs})}{f(l_{rs})}\frac{\partial f(l_{rs})}{\partial l_{rs}} =$$

$$\max_{l_{rs}\in\left[\left(\frac{1}{B}\frac{1-\beta}{1-\beta\alpha}\right)^{\frac{1}{\beta}},1\right]} -\left[\frac{(l_{rs})^{1-\beta}\left[(1-\beta)\left(1-B\,(l_{rs})^{\beta}\right)(1-l_{rs})+l_{rs}\left(1-B\,(l_{rs})^{\beta}\right)\right]}{(l_{rs})^{1-\beta}-\frac{\theta B}{(1-\alpha)}\tau(1-l_{rs})}+\right.$$

$$\left.\frac{1-\alpha}{\alpha}\left[(1-\beta)\left(1-B\,(l_{rs})^{\beta}\right)(1-l_{rs})-\beta B\,(l_{rs})^{\beta}\,(1-l_{rs})\right]\right]$$

Note that the objective function is a continuous function for $l_{rs} > l_{rs}^{\min}$, given that the objective function is continuous in the range in which it is maximized. Furthermore, because $B \geq \frac{1-\beta}{1-\beta\alpha}$ the correspondence $l_{rs} \in \left[\left(\frac{1}{B}\frac{1-\beta}{1-\beta\alpha}\right)^{\frac{1}{\beta}}, 1\right]$ is also continuous. Thus, the Maximum Theorem implies that $\phi(B)$ is a continuous function. Furthermore:

$$\begin{aligned}
&\phi\left(\frac{1-\beta}{1-\beta\alpha}\right) \ = \\
&\max_{l_{rs}\in[1,1]} - \left[\frac{(l_{rs})^{1\text{-}\beta}\left[(1\text{-}\beta)\left(1\text{-}\left(\frac{1\text{-}\beta}{1\text{-}\beta\alpha}\right)(l_{rs})^{\beta}\right)(1\text{-}l_{rs}) + l_{rs}\left(1\text{-}\left(\frac{1\text{-}\beta}{1\text{-}\beta\alpha}\right)(l_{rs})^{\beta}\right)\right]}{(l_{rs})^{1\text{-}\beta}\text{-}\frac{\theta B}{(1-\alpha)}\tau(1-l_{rs})} + \right. \\
&\left.\frac{1\text{-}\alpha}{\alpha}\left[(1\text{-}\beta)\left(1\text{-}\left(\frac{1\text{-}\beta}{1\text{-}\beta\alpha}\right)(l_{rs})^{\beta}\right)(1\text{-}l_{rs}) - \beta\left(\frac{1\text{-}\beta}{1\text{-}\beta\alpha}\right)(l_{rs})^{\beta}(1\text{-}l_{rs})\right]\right] \ = \\
&-\left(1-\frac{1-\beta}{1-\beta\alpha}\right) \ < \ 0
\end{aligned}$$

Thus, it follows from the continuity of $\phi(B)$ that either $\forall B \leq 1 : \phi(B) < 0$ or $\exists\overline{B} \in \left(\frac{1-\beta}{1-\beta\alpha}, 1\right]$ such that $\forall B < \overline{B} : \phi(B) < 0$ and if $B = \overline{B} : \phi(B) = 0$.

Remark 2. *When $B < \overline{B}$ the function $f(l_{rs})$ is strictly decreasing, which also means that the inverse of such function, $\widehat{l}_{rs}(\widetilde{k}) = f^{-1}(\widetilde{k})$, is also strictly decreasing.*

Remark 3. *If $\frac{1-\alpha}{\alpha} > 1$ then $\overline{B} < 1$.*

To prove this, let's define the function $g(l_{rs})$ as follows:

$$\begin{aligned}
g(l_{rs}, B) \ &= \ \frac{\left(1 - B\,(l_{rs})^{\beta}\right) l_{rs}}{f(l_{rs}, B)}\frac{\partial f(l_{rs}, B)}{\partial l_{rs}} \ = \\
&- \left[\frac{(l_{rs})^{1-\beta}\left[(1-\beta)\left(1 - B\,(l_{rs})^{\beta}\right) + \frac{l_{rs}\left(1-B(l_{rs})^{\beta}\right)}{(1-l_{rs})}\right]}{(l_{rs})^{1-\beta} - \frac{\theta B}{(1-\alpha)}\tau(1-l_{rs})} + \right. \\
&\left.\frac{1-\alpha}{\alpha}\left[(1-\beta)\left(1 - B\,(l_{rs})^{\beta}\right) - \beta B\,(l_{rs})^{\beta}\right]\right]
\end{aligned}$$

Note that

$$\lim_{B\to 1} g(B^{\gamma}, B) \;=\; -\lim_{B\to 1}\left[\frac{B^{\gamma(1-\beta)}\left[(1-\beta)\left(1-B^{1+\gamma\beta}\right)+\frac{B^{\gamma}\left(1-B^{1+\gamma\beta}\right)}{(1-B^{\gamma})}\right]}{B^{\gamma(1-\beta)}-\frac{\theta B}{(1-\alpha)}\tau(1-B^{\gamma})}+\frac{1-\alpha}{\alpha}\left[(1-\beta)\left(1-B^{1+\gamma\beta}\right)-\beta B^{1+\gamma\beta}\right]\right] \;=\; -\left[\frac{1+\gamma\beta}{\gamma\beta}-\frac{1-\alpha}{\alpha}\right]\beta$$

where $\gamma \in \Re_{++}$. Because $\lim_{\gamma\to+\infty}\frac{1+\gamma\beta}{\gamma\beta} = 1$, if $\frac{1-\alpha}{\alpha} > 1$ then a large enough $\widetilde{\gamma}$ always exists such that:

$$\lim_{B\to 1} g(B^{\widetilde{\gamma}}, B) \;=\; -\left[\frac{1+\widetilde{\gamma}\beta}{\widetilde{\gamma}\beta}-\frac{1-\alpha}{\alpha}\right]\beta \;>\; 0$$

Thus, there exist $\widetilde{B} < 1$ such that $g\left(\widetilde{B}^{\widetilde{\gamma}}, \widetilde{B}\right) > 0$. It follows from the definition of $g(l_{rs}, B)$ that:

$$\frac{\partial f\left(\widetilde{B}^{\widetilde{\gamma}}, \widetilde{B}\right)}{\partial l_{rs}} > 0$$

Thus: $\overline{B} < \widetilde{B} < 1$.

8.5. Proof proposition 5.2

It follows from (3.18) that:

$$\lim_{a\to+\infty} l_{rs}\,(\tau{+}a) = 1$$

Thus:

$$\lim_{a\to+\infty} v^{S} \;=\; \lim_{a\to+\infty}\frac{1}{\sigma}\left(\alpha A^{\frac{1}{\alpha}}\left[\frac{(1\text{-}\alpha)}{\theta B}\left(l_{rs}\,(\tau{+}a)\right)^{1-\beta}\left[1-B\left(l_{rs}\,(\tau{+}a)\right)^{\beta}\right]\right]^{\frac{1-\alpha}{\alpha}}-\delta-\rho\right) \;=\; \frac{1}{\sigma}\left(\alpha A^{\frac{1}{\alpha}}\left[\frac{(1\text{-}\alpha)}{\theta B}\left[1-B\right]\right]^{\frac{1-\alpha}{\alpha}}-\delta-\rho\right)$$

Given that the function that relates the growth rate in the *South* with the ratio public revenues/income shows a hump-shaped relationship and the growth rate at the *South* should be equal to the growth rate at the *North* at BGP, the bad BGP exists if and only if $\lim_{a\to+\infty} v^S < v^N$. This last condition may be rewritten as follows:

$$\frac{1}{\sigma}\left(\alpha A^{\frac{1}{\alpha}}\left[\frac{(1\text{-}\alpha)}{\theta B}\left[1-B\right]\right]^{\frac{1-\alpha}{\alpha}}-\delta-\rho\right) < v^N \Leftrightarrow B > \underline{B} \equiv \frac{(1\text{-}\alpha)\left(\alpha^{\alpha}A\right)^{\frac{1}{1-\alpha}}}{(1\text{-}\alpha)\left(\alpha^{\alpha}A\right)^{\frac{1}{1-\alpha}}+\theta\left(\sigma v^N+\delta+\rho\right)^{\frac{\alpha}{1-\alpha}}}$$

8.6. Linearized dynamic system

If we linearize the dynamic system (5.5)-(5.6) around the steady state, we get:

$$\begin{bmatrix} \dot{\widetilde{c}}(t) \\ \dot{\widetilde{k}}(t) \end{bmatrix} = \begin{bmatrix} 0 & \frac{\partial v_{\widetilde{c}}\left(\widetilde{k}^{BGP}\right)}{\partial \widetilde{k}}\widetilde{c}^{BGP} \\ -1 & a_{22}^{BGP} \end{bmatrix}\begin{bmatrix} \widetilde{c}(t)-\widetilde{c}^{BGP} \\ \widetilde{k}(t)-\widetilde{k}^{BGP} \end{bmatrix}$$

where $v_{\widetilde{c}}$ is the growth rate of $\widetilde{c}$:

$$v_{\widetilde{c}}\left(\widetilde{k}\right) = \frac{1}{\sigma}\left(r\left(\widetilde{k}\right)-\rho\right)-v^N$$

a_{22}^{BGP} is defined as follows:

$$\begin{aligned}
a_{22}^{BGP} &= \frac{1}{\alpha}\left[r(\widetilde{k}^{BGP})+\frac{\partial r\left(\widetilde{k}^{BGP}\right)}{\partial \widetilde{k}}\widetilde{k}^{BGP}\right]\left(1-\tau(1-B\left(l_{rs}^{BGP}\right)^{\beta})\right)+ \\
&\left[\tau\widetilde{y}\left(\widetilde{k}(t)\right)+\psi\right]\beta B\left(\widehat{l}_{rs}^{BGP}\right)^{\beta-1}\frac{\partial \widehat{l}_{rs}\left(\widetilde{k}^{BGP}\right)}{\partial \widetilde{k}}-(\delta+v^N) = \\
&\frac{1}{\alpha}\left[\sigma v^N+\rho+\frac{\partial r\left(\widetilde{k}^{BGP}\right)}{\partial \widetilde{k}}\widetilde{k}^{BGP}\right]\left(1-\tau(1-B\left(l_{rs}^{BGP}\right)^{\beta})\right)+ \\
&\left[\tau\widetilde{y}\left(\widetilde{k}(t)\right)+\psi\right]\beta B\left(\widehat{l}_{rs}^{BGP}\right)^{\beta-1}\frac{\partial \widehat{l}_{rs}\left(\widetilde{k}^{BGP}\right)}{\partial \widetilde{k}}-\left(\delta+v^N\right)
\end{aligned}$$

The characteristic equation associated with the above linear dynamic system is as follows:

$$\begin{vmatrix} -\lambda & \frac{\partial v_{\widetilde{c}}\left(\widetilde{k}^{BGP}\right)}{\partial \widetilde{k}}\widetilde{c}^{BGP} \\ -1 & a_{22}^{BGP}-\lambda \end{vmatrix} = 0$$

$$\lambda^2+a_{22}^{BGP}\lambda+\frac{\partial v_{\widetilde{c}}\left(\widetilde{k}^{BGP}\right)}{\partial \widetilde{k}}\widetilde{c}^{BGP} = 0$$

The roots associated with the above linear dynamic system are as follows:

$$\lambda = \frac{-a_{22}^{BGP} \pm \sqrt{(a_{22}^{BGP})^2 - 4\frac{\partial v_{\widetilde{c}}\left(\widetilde{k}^{BGP}\right)}{\partial \widetilde{k}}\widetilde{c}^{BGP}}}{2} \tag{8.10}$$

It follows from lemma 5.1, equation (8.8) and assumptions (5.1), (5.2) and (5.3) that:

$$\begin{aligned}
&\lim_{\widetilde{k}\to+\infty} \widetilde{y}\left(\widetilde{k}\right) = \lim_{\widetilde{k}\to+\infty} A^{\frac{1}{\alpha}} \left[\frac{(1\text{-}\alpha)}{\theta B}\left(\widehat{l}_{rs}\left(\widetilde{k}\right)\right)^{1-\beta}\left[1\text{-}B\left(\widehat{l}_{rs}\left(\widetilde{k}\right)\right)^{\beta}\right]\right]^{\frac{1-\alpha}{\alpha}} \widetilde{k} = \\
&\lim_{\widetilde{k}\to+\infty} A^{\frac{1}{\alpha}} \left[\frac{(1\text{-}\alpha)}{\theta B}\left(l_{rs}^{\min}\right)^{1-\beta}\left[1\text{-}B\left(l_{rs}^{\min}\right)^{\beta}\right]\right]^{\frac{1-\alpha}{\alpha}} \widetilde{k} = +\infty \Rightarrow \\
&\lim_{\widetilde{k}\to+\infty} \left[\tau + \frac{\psi}{\widetilde{y}\left(\widetilde{k}\right)}\right] = \tau \Rightarrow \lim_{\widetilde{k}\to+\infty} v_{\widetilde{c}}\left(\widetilde{k}\right) = v_{\text{no aid}}^{S} - v^{N} < 0
\end{aligned} \tag{8.11}$$

Moreover, $\lim_{\widetilde{k}\to 0}\widetilde{y}\left(\widetilde{k}\right) = 0$.

Remark, $\widehat{l}_{rs}\left(\widetilde{k}\right) \overset{Def}{\Leftrightarrow} \frac{\left(\widehat{l}_{rs}\left(\widetilde{k}\right)\right)^{1-\beta}}{1-\widehat{l}_{rs}\left(\widetilde{k}\right)} = \frac{\theta B}{(1-\alpha)}\left[\tau + \frac{\psi}{\widetilde{y}\left(\widetilde{k}\right)}\right]$. Thus,

$$\begin{aligned}
&\lim_{\widetilde{k}\to 0}\widetilde{y}\left(\widetilde{k}\right) = 0 \Rightarrow \lim_{\widetilde{k}\to 0}\frac{\theta B}{(1-\alpha)}\left[\tau + \frac{\psi}{\widetilde{y}\left(\widetilde{k}\right)}\right] = +\infty \Rightarrow \\
&\Rightarrow \lim_{\widetilde{k}\to 0}\widehat{l}_{rs}\left(\widetilde{k}\right) = 1 \Rightarrow \\
&\Rightarrow \lim_{\widetilde{k}\to 0} r\left(\widetilde{k}\right) = \alpha A^{\frac{1}{\alpha}}\left[\frac{(1\text{-}\alpha)}{\theta B}[1\text{-}B]\right]^{\frac{1-\alpha}{\alpha}} - \delta \Rightarrow \\
&\Rightarrow \lim_{\widetilde{k}\to 0} v_{\widetilde{c}}\left(\widetilde{k}\right) = \frac{1}{\sigma}\left(\alpha A^{\frac{1}{\alpha}}\left[\frac{(1\text{-}\alpha)}{\theta B}[1\text{-}B]\right]^{\frac{1-\alpha}{\alpha}} - \delta\text{–}\rho\right) - v^{N} < 0
\end{aligned} \tag{8.12}$$

$$\widetilde{k}^{*} \overset{def}{\Leftrightarrow} \frac{\psi}{\widetilde{y}\left(\widetilde{k}^{*}\right)} = a^{*} \Rightarrow v_{\widetilde{c}}\left(\widetilde{k}^{*}\right) = \max_{\frac{T}{y}} v^{S} - v^{N} > 0 \tag{8.13}$$

Thus: (i) when $\widetilde{k}$ is small enough $v_{\widetilde{c}}\left(\widetilde{k}\right) < 0$; (ii) when $\widetilde{k}$ is large enough $v_{\widetilde{c}}\left(\widetilde{k}\right) < 0$; (iii) There are values of $\widetilde{k}$ (for instance, $\widetilde{k}^{*}$) in which $v_{\widetilde{c}}\left(\widetilde{k}\right) > 0$; (iv) we know that there are two BGPs, consequently $v_{\widetilde{c}}\left(\widetilde{k}\right) = 0$ for two values $\widetilde{k}^{bad}$ and $\widetilde{k}^{good}$, where

$\widetilde{k}^{bad} < \widetilde{k}^{good}$. Thus, when $\widetilde{k}$ increases $v_{\widetilde{c}}\left(\widetilde{k}\right)$ is negative for small values of $\widetilde{k}$ ($\widetilde{k}$ smaller than $\widetilde{k}^{bad}$), then it becomes zero when $\widetilde{k} = \widetilde{k}^{bad}$, then it becomes positive, then zero again when $\widetilde{k} = \widetilde{k}^{good}$, and finally, negative again for levels of $\widetilde{k}$ higher than $\widetilde{k}^{good}$. Thus, generically:

$$\begin{aligned}\frac{\partial v_{\widetilde{c}}\left(\widetilde{k}^{bad}\right)}{\partial \widetilde{k}} &> 0\\ \frac{\partial v_{\widetilde{c}}\left(\widetilde{k}^{good}\right)}{\partial \widetilde{k}} &< 0\end{aligned}$$

Let's consider first the good BGP. We now know that $\sqrt{(a_{22}^{good})^2 - 4\frac{\partial v_{\widetilde{c}}\left(\widetilde{k}^{good}\right)}{\partial \widetilde{k}}\widetilde{c}^{good}} > \left|a_{22}^{good}\right| > 0$. Thus, according to equation (8.10) the roots of the linearized linear system in the surrounding of the good BGP are real ones, one of them being negative and the other positive, which means that the good BGP is a saddle point.

Let's turn now to the bad BGP. We have four possible cases:

1. $(a_{22}^{bad})^2 - 4\frac{\partial v_{\widetilde{c}}\left(\widetilde{k}^{bad}\right)}{\partial \widetilde{k}}\widetilde{c}^{bad} > 0$ and $a_{22}^{bad} > 0$, both roots are real and negative, then the bad BGP is a source.

2. $(a_{22}^{bad})^2 - 4\frac{\partial v_{\widetilde{c}}\left(\widetilde{k}^{bad}\right)}{\partial \widetilde{k}}\widetilde{c}^{bad} > 0$ and $a_{22}^{bad} < 0$, both roots are real and positive, then the bad BGP is a focus.

3. $(a_{22}^{bad})^2 - 4\frac{\partial v_{\widetilde{c}}\left(\widetilde{k}^{bad}\right)}{\partial \widetilde{k}}\widetilde{c}^{bad} < 0$ and $a_{22}^{bad} > 0$, both roots are complex, and the real part of the root $-a_{22}^{bad}$ is negative, then the bad BGP is a source.

4. $(a_{22}^{bad})^2 - 4\frac{\partial v_{\widetilde{c}}\left(\widetilde{k}^{bad}\right)}{\partial \widetilde{k}}\widetilde{c}^{bad} < 0$ and $a_{22}^{bad} < 0$, both roots are complex, and the real part of the root $-a_{22}^{bad}$ is positive, then the bad BGP is a focus.

8.7. Transitional dynamics: Bounded international aid

We have defined l_{rs}^{*} as the amount of time devoted to rent-seeking which maximizes the growth rate of consumption:

$$l_{rs}^{*} = \arg\max \frac{1}{\sigma}\left(\alpha A^{\frac{1}{\alpha}}\left[\frac{(1\text{-}\alpha)}{\theta B}(l_{rs})^{1-\beta}\left[1\text{-}B(l_{rs})^{\beta}\right]\right]^{\frac{1-\alpha}{\alpha}} - \delta - \rho\right) \Rightarrow$$

$$\frac{\partial v_c}{\partial l_{rs}} = \frac{\partial\left[\frac{1}{\sigma}\left(\alpha A^{\frac{1}{\alpha}}\left[\frac{(1-\alpha)}{\theta B}(l_{rs}^{*})^{1-\beta}\left[1\text{-}B(l_{rs}^{*})^{\beta}\right]\right]^{\frac{1-\alpha}{\alpha}} - \delta - \rho\right)\right]}{\partial l_{rs}} =$$

$$\frac{1}{\sigma}\alpha A^{\frac{1}{\alpha}}\left[\frac{(1\text{-}\alpha)}{\theta B}(l_{rs}^{*})^{1\text{-}\beta}\left[1\text{-}B(l_{rs}^{*})^{\beta}\right]\right]^{\frac{1\text{-}\alpha}{\alpha}}\frac{1\text{-}\alpha}{\alpha}\frac{1}{l_{rs}^{*}}\left[(1\text{-}\beta) - \frac{\beta B(l_{rs}^{*})^{\beta}}{1\text{-}B(l_{rs}^{*})^{\beta}}\right] = 0$$

Because the function $\frac{\beta B(l_{rs})^{\beta}}{1-B(l_{rs})^{\beta}}$ is increasing, it follows that :

$$\forall l_{rs} < l_{rs}^{*} \quad \left[(1\text{-}\beta)\text{-}\frac{\beta B(l_{rs})^{\beta}}{1\text{-}B(l_{rs})^{\beta}}\right] > \left[(1\text{-}\beta)\text{-}\frac{\beta B(l_{rs}^{*})^{\beta}}{1\text{-}B(l_{rs}^{*})^{\beta}}\right] = 0 \Rightarrow$$

$$\frac{\partial v_c}{\partial l_{rs}} = \frac{\partial\left[\frac{1}{\sigma}\left(\alpha A^{\frac{1}{\alpha}}\left[\frac{(1-\alpha)}{\theta B}(l_{rs})^{1-\beta}\left[1\text{-}B(l_{rs})^{\beta}\right]\right]^{\frac{1-\alpha}{\alpha}} - \delta - \rho\right)\right]}{\partial l_{rs}} =$$

$$\frac{1}{\sigma}\alpha A^{\frac{1}{\alpha}}\left[\frac{(1\text{-}\alpha)}{\theta B}(l_{rs})^{1\text{-}\beta}\left[1\text{-}B(l_{rs})^{\beta}\right]\right]^{\frac{1\text{-}\alpha}{\alpha}}\frac{1\text{-}\alpha}{\alpha}\frac{1}{l_{rs}}\left[(1\text{-}\beta)\text{-}\frac{\beta B(l_{rs})^{\beta}}{1\text{-}B(l_{rs})^{\beta}}\right] > 0 \quad (8.14)$$

In this case the amount of labor devoted to predation is bounded above:

$$l_{rs}(t) = \min\left\{\widehat{l}_{rs}\left(\widetilde{k}\right), l_{rs}^{*}\right\}, \text{ where: } l_{rs}^{*} \overset{Def}{\Leftrightarrow} \frac{(l_{rs}^{*})^{1-\beta}}{1-l_{rs}^{*}} = \frac{\theta B}{(1-\alpha)}[\tau + a^{*}]$$

Let's define $\widetilde{k}^{*}$ as follows:

$$\widetilde{k}^{*} \overset{def}{\Leftrightarrow} \frac{\psi}{\widetilde{y}\left(\widetilde{k}^{*}\right)} = a^{*} \Rightarrow v_{\widetilde{c}}\left(\widetilde{k}^{*}\right) = \max_{\frac{T}{y}} v^{S} - v^{N} > 0$$

If $\widetilde{k} \leq \widetilde{k}^{*}$:

$$l_{rs}(t) = \min\left\{\widehat{l}_{rs}\left(\widetilde{k}\right), l_{rs}^{*}\right\} = l_{rs}^{*} \Rightarrow v_{\widetilde{c}}\left(\widetilde{k}\right) = v_{\widetilde{c}}\left(\widetilde{k}^{*}\right) = \max_{\frac{T}{y}} v^{S} - v^{N} > 0$$

If $\widetilde{k} > \widetilde{k}^{*}$:

$$l_{rs}(t) = \min\left\{\widehat{l}_{rs}\left(\widetilde{k}\right), l_{rs}^{*}\right\} = \widehat{l}_{rs}\left(\widetilde{k}\right) < l_{rs}^{*}$$

Thus, when $\widetilde{k} > \widetilde{k}^{*}$, we know that $l_{rs} < l_{rs}^{*}$. Then it follows from equation (8.14) that

$$\frac{\partial v_c}{\partial l_{rs}} = \frac{\partial \left[\frac{1}{\sigma}\left(\alpha A^{\frac{1}{\alpha}}\left[\frac{(1-\alpha)}{\theta B}\left(l_{rs}\right)^{1-\beta}\left[1-B\left(l_{rs}\right)^{\beta}\right]\right]^{\frac{1-\alpha}{\alpha}} - \delta - \rho\right)\right]}{\partial l_{rs}} > 0 \Rightarrow$$

$$\frac{\partial \left[\alpha A^{\frac{1}{\alpha}}\left[\frac{(1-\alpha)}{\theta B}\left(l_{rs}\right)^{1-\beta}\left[1-B\left(l_{rs}\right)^{\beta}\right]\right]^{\frac{1-\alpha}{\alpha}}\right]}{\partial l_{rs}} > 0 \Rightarrow$$

$$\frac{\partial r(\widetilde{k})}{\partial \widetilde{k}} = \frac{\partial \left[\alpha A^{\frac{1}{\alpha}}\left[\frac{(1\text{-}\alpha)}{\theta B}\left(\widehat{l}_{rs}\left(\widetilde{k}(t)\right)\right)^{1\text{-}\beta}\left[1\text{-}B\left(\widehat{l}_{rs}\left(\widetilde{k}(t)\right)\right)^{\beta}\right]\right]^{\frac{1\text{-}\alpha}{\alpha}}\right]}{\partial l_{rs}} \frac{\partial \widehat{l}_{rs}\left(\widetilde{k}(t)\right)}{\partial \widetilde{k}} < 0$$

This means that the interest rate is constant for $\widetilde{k} \leq \widetilde{k}^{*}$ (with the growth rate of $\widetilde{c}$ being positive) and strictly decreasing for $\widetilde{k} > \widetilde{k}^{*}$. This means that a unique BGP exists such that $\widetilde{k}^{BGP} > \widetilde{k}^{*}$. Given that $\widetilde{k}^{BGP} > \widetilde{k}^{*}$, then $\frac{\partial r\left(\widetilde{k}^{BGP}\right)}{\partial \widetilde{k}} < 0$. As it was shown for the case in which international aid is proportional to the income of the *North*, when $\frac{\partial r\left(\widetilde{k}^{BGP}\right)}{\partial \widetilde{k}} < 0$ the BGP is a saddle point.

8.8. Proof of Proposition 6.1

Lemma 8.1. *If one of the following sufficient conditions hold:*
$*$ *Sufficient condition 1:* $(1-B) < \left(l_{rs}\left(\tau-\psi\right)\right)^{1-\beta}\left[1-B\left(l_{rs}\left(\tau-\psi\right)\right)^{\beta}\right]$ *and* $\tau < a^{*}$
$*$ *Sufficient condition 2:* $(1-B) > \left(l_{rs}\left(\tau-\psi\right)\right)^{1-\beta}\left[1-B\left(l_{rs}\left(\tau-\psi\right)\right)^{\beta}\right]$ *and* $\tau > a^{*}$
then there are at least three BGPs, with only one of them being symmetric, $\widetilde{y} = 1$. *Moreover, the growth rate along the symmetric BGP is higher than the growth rate in any of the asymmetric BGPs.*

8.8.1. Proof Lemma 8.1:

Sufficient condition 1: $(1-B) < \left(l_{rs}\left(\tau-\psi\right)\right)^{1-\beta}\left[1-B\left(l_{rs}\left(\tau-\psi\right)\right)^{\beta}\right]$ and $\tau < a^{*}$

The following condition should hold on the BGP (see equation 6.2):

$$F(\widetilde{y}) = v\left(\tau + \psi\left(\frac{1}{\widetilde{y}} - 1\right)\right) - v\left(\tau + \psi\left(\widetilde{y} - 1\right)\right) = 0$$

Let's define $\widetilde{y}^{*}$ as the ratio income of the *South* income of the *North*, which maximizes the growth rate in the *South*:

$$\widetilde{y}^* \overset{def}{\Leftrightarrow} \arg\max v\left(\tau{+}\psi\left(\frac{1}{\widetilde{y}^*}\text{-}1\right)\right) \quad \Leftrightarrow \quad \tau{+}\psi\left(\frac{1}{\widetilde{y}^*}\text{-}1\right) = a^* \Leftrightarrow \widetilde{y}^* = \frac{\psi}{\psi{+}a^*\text{-}\tau} < 1$$
$$\tau + \psi\left(\widetilde{y}^* - 1\right) \;=\; a^* - \left[1 + \frac{\psi}{\psi + (a^* - \tau)}\right](a^* - \tau) < a^*$$

where a^* is defined in equation (4.3) as the level of international aid that maximizes the growth rate of the *South*. It follows from the last equation and the assumption of $\tau < a^*$ that:

$$F\left(\widetilde{y}^*\right) \;=\; v(a^*) - v\left(\tau + \psi\left(\widetilde{y}^* - 1\right)\right) \;>\; v(a^*) - v\left(\tau\right) \;>\; 0 \tag{8.15}$$

It follows from (3.18) that:

$$\begin{aligned}
&\lim_{\widetilde{y}\to 0} l_{rs}\left(\tau + \psi\left(\frac{1}{\widetilde{y}} - 1\right)\right) \;=\; \lim_{\frac{T}{y}\to+\infty} l_{rs}\left(\frac{T}{y}\right) \;=\; 1 \Rightarrow \\
&\lim_{\widetilde{y}\to 0} v\left(\tau + \psi\left(\frac{1}{\widetilde{y}} - 1\right)\right) \;=\; \frac{1}{\sigma}\left(\alpha A^{\frac{1}{\alpha}}\left[\frac{(1-\alpha)}{\theta B}\left[1 - B\right]\right]^{\frac{1-\alpha}{\alpha}} - \delta - \rho\right) \;< \\
&\frac{1}{\sigma}\left(\alpha A^{\frac{1}{\alpha}}\left[\frac{(1-\alpha)}{\theta B}\left(l_{rs}\left(\tau - \psi\right)\right)^{1-\beta}\left[1 - B\left(l_{rs}\left(\tau - \psi\right)\right)^{\beta}\right]\right]^{\frac{1-\alpha}{\alpha}} - \delta - \rho\right) \;= \\
&\lim_{\widetilde{y}\to 0} v\left(\tau + \psi\left(\widetilde{y} - 1\right)\right) \;\Rightarrow\; \lim_{\widetilde{y}\to 0} F\left(\widetilde{y}\right) \;<\; 0
\end{aligned} \tag{8.16}$$

where we have used the assumption that (1-B) $<$ $\left(l_{rs}\left(\tau\text{-}\psi\right)\right)^{1-\beta}\left[1 - B\left(l_{rs}\left(\tau\text{-}\psi\right)\right)^{\beta}\right]$. Thus, it follows from equations (8.15), (8.16) and the continuity of $F(.)$ that $\widetilde{y}^{asymmetric,\ BGP1} \in (0, \widetilde{y}^*)$ exists such that $F(\widetilde{y}^{asymmetric,\ BGP1}) = 0$. Furthermore, $\widetilde{y}^{asymmetric,\ BGP1} < \widetilde{y}^* = \frac{\psi}{\psi + a^* - \tau} < 1$, i.e., an asymmetric BGP exists such that $\widetilde{y}^{asymmetric,\ BGP1} < 1$.

Then, it is easy to see that:

$$\begin{aligned}
F(\widetilde{y}) \;&=\; v\left(\tau + \psi\left(\frac{1}{\widetilde{y}} - 1\right)\right) - v\left(\tau + \psi\left(\widetilde{y} - 1\right)\right) \;= \\
&-v\left(\tau + \psi\left(\frac{1}{\frac{1}{\widetilde{y}}} - 1\right)\right) + v\left(\tau + \psi\left(\frac{1}{\widetilde{y}} - 1\right)\right) \;=\; -F\left(\frac{1}{\widetilde{y}}\right)
\end{aligned}$$

Let's define $\widetilde{y}^{asymmetric,\ BGP2} \equiv \frac{1}{\widetilde{y}^{asymmetric,\ BGP1}}$. It follows from the above equation that $F\left(\widetilde{y}^{asymmetric,\ BGP2}\right) = F\left(\frac{1}{\widetilde{y}^{asymmetric,\ BGP1}}\right) = 0$. So, $\widetilde{y}^{asymmetric,\ BGP2}$ is another BGP. Because $\widetilde{y}^{asymmetric,\ BGP1} < 1$ then $\widetilde{y}^{asymmetric,\ BGP2} \equiv \frac{1}{\widetilde{y}^{asymmetric,\ BGP1}} > 1$.

Afterwards, if follows from the definition of $F(.)$ that $\widetilde{y} = 1$ is a BGP, that is, the symmetric BGP, $\widetilde{y}^{symmetric\ BGP} = 1$.

Finally, note that:

$$\widetilde{y}^{asymmetric,\ BGP1} < 1 \Rightarrow v\left(\tau + \psi\left(\widetilde{y}^{asymmetric,\ BGP1} - 1\right)\right) < v(\tau)$$

where we are using the assumption that $\tau < a^*$ and the fact that $v\left(\frac{T}{y}\right)$ is increasing when $\frac{T}{y} < a^*$.

Sufficient condition 2: $(1 - B) > (l_{rs}(\tau - \psi))^{1-\beta}\left[1 - B\left(l_{rs}(\tau - \psi)\right)^{\beta}\right]$ and $\tau > a^*$

Let's define $\widetilde{y}^*$ as the ratio income of the *South* income of the *North* which maximizes the growth rate in the *South*:

$$\widetilde{y}^* \overset{def}{\Leftrightarrow} \arg\max v\left(\tau + \psi\left(\frac{1}{\widetilde{y}^*} - 1\right)\right)$$

If we now define function $F(\widetilde{y})$ as follows:

$$F(\widetilde{y}) \;=\; v\left(\tau + \psi\left(\widetilde{y} - 1\right)\right) - v\left(\tau + \psi\left(\frac{1}{\widetilde{y}} - 1\right)\right) \;=\; 0$$

and we assume alternatively that $\tau > a^*$ and $(1\text{-}B) > (l_{rs}(\tau\text{-}\psi))^{1\text{-}\beta}\left[1\text{-}B\left(l_{rs}(\tau\text{-}\psi)\right)^{\beta}\right]$, then it is easy to see that rest of the proof is identical to the previous case.

8.8.2. Proof Proposition 6.1

The sufficient condition 1 of the lemma 8.1 may be rewritten as follows:

$$\begin{aligned} B \;&>\; \frac{1 - (l_{rs}(\tau - \psi))^{1-\beta}}{1 - l_{rs}(\tau - \psi)} \;=\; \frac{1}{1 - l_{rs}(\tau - \psi)} - \frac{\theta B}{(1 - \alpha)}(\tau - \psi) \;\Leftrightarrow \\ F(B) \;&=\; B\left[1 + \frac{\theta}{(1 - \alpha)}(\tau - \psi)\right] - \frac{1}{1 - l_{rs}(\tau - \psi)} \;>\; 0 \end{aligned}$$

where we have used definition of $(l_{rs}(\tau - \psi))$ in equation (3.18).

Let's define $\widehat{B} \overset{Def}{\Leftrightarrow} F(\widehat{B}) \;=\; 0$. Note that:

$$\frac{\partial F(B)}{\partial B} = 1 + \frac{\theta}{(1 - \alpha)}(\tau - \psi)\frac{(1 - \beta) + \beta l_{rs} - l_{rs}^{\beta}}{(1 - \beta)\left[1 - l_{rs}\right] + l_{rs}}$$

Taking account that $\min_{l_{rs}\in[0,1]}\left[(1-\beta)+\beta l_{rs}-l_{rs}^{\beta}\right] = [(1-\beta)+\beta-1] = 0$, it follows that:

$$\frac{\partial F(B)}{\partial B} = 1 + \frac{\theta}{(1-\alpha)}(\tau-\psi)\frac{(1-\beta)+\beta l_{rs}-l_{rs}^{\beta}}{(1-\beta)[1-l_{rs}]+l_{rs}} \geq 1 > 0$$

Thus, if $B > \widehat{B}$, then $F(B) > 0$ and sufficient condition in lemma 8.1 is satisfied.

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